ADVANCED RECONSTRUCTION TECHNIQUES IN MRI - 1

Presented by
Rahil Kothari

IMAGE RECONSTRUCTION IN MRI

The signal recorded in MRI is given by the Signal Equation,

$$s(t) = \int_x \int_y m(x,y) e^{-i2\pi[k_x(t)x + k_y(t)y]} dxdy \qquad ...(1)$$

This is basically a Fourier Transform of the image to be acquired m(x,y).

Hence a simple Inverse Fourier Transform would yield our required image.

• Magnetic Resonance Imaging is increasingly being used for fast imaging applications such as real-time cardiac imaging, functional brain imaging, etc.

• Imaging speed in MRI is mainly limited by different imaging parameters selected by the pulse sequences, the subject being imaged and the RF hardware system in operation.

• New pulse sequences have been developed in order to decrease the imaging time by a faster k-space scan.

• However, they may not be fast enough to facilitate imaging in real time.

• Thus, image reconstruction techniques have been developed, in order to enable faster acquisition in MRI.

Among these are,

a. Parallel Imaging Reconstruction

b. Partial Fourier Reconstruction

PARALLEL-IMAGING RECONSTRUCTION

WHAT IS PARALLEL-IMAGING?

• Parallel Imaging, is a technique, that uses phased array coils, for the purpose of faster scanning.

WHAT ARE PHASED ARRAY COILS?

• Phased array coils, as discovered by Roemer et al. in 1990, consist of a number of mutually decoupled surface coils that simultaneously receive MR signals.

HOW DOES SNR IMROVE WITH MULTIPLE COILS?

- The noise in the MR signal is predominantly from the patient rather than from the coils and electronics.
- The noise in a single coil is induced by sources within the patient and is weighted by the spatial sensitivity profile.
- Phased array coils improve SNR by reducing the coil size, thereby reducing the amplitude of noise detected.
- Therefore, multiple small coils, are combined together in such a way so as to cover the same volume as a single large area by having their volumes overlap slightly.
- Thus these coils yield a signal with the same amplitude as a single coil, with the noise greatly reduced, thereby improving SNR.

HOW DO THEY REDUCE SCAN TIME?

- Scan Time is proportional to the number of Phase encoding lines acquired.
- If we increase the distance between these lines by a factor of R, while keeping spatial resolution fixed, the scan time reduces by the same factor. In parallel imaging, R is called the *reduction factor* or *acceleration factor*.
- Increasing the distance between phase-encoding lines also decreases FOV. If the object extends outside the reduced FOV, aliasing occurs as seen in Fig. 1.

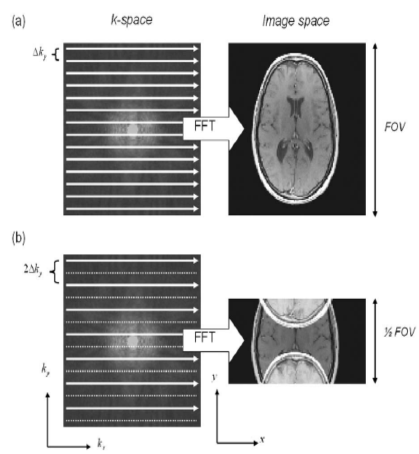


Figure 1. (a) Conventional acquisition of fully sampled kspace, resulting in a full FOV image after Fourier transformation.
(b) Undersampled acquisition (R = 2), resulting in a reduced FOV (FOV/2) with aliasing artifacts. Solid lines indicate acquired k-space lines, dashed lines indicate non-acquired k-space lines. [5]

• In parallel imaging, the spatial dependence of the B1 field of the receiver array coil is used to remove or prevent this aliasing.

• There are two basic methods used to avoid aliasing in Parallel Imaging. They are,

1. SENSE
2. SMASH

SENSE (Sensitivity encoding)

In SENSE, the individual receive coil k-space sets are separately Fourier transformed, resulting in aliased images. These aliased images are then combined using weights constructed from their sensitivities to give a single final image with the aliasing artifacts removed.

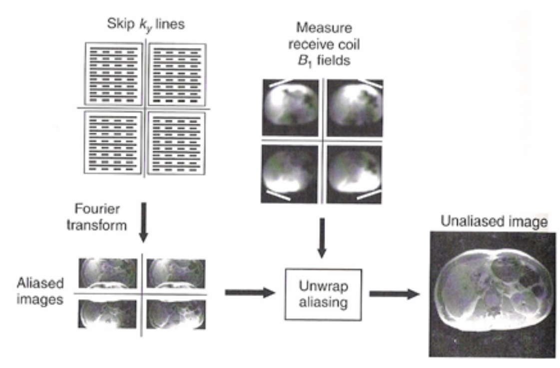


Figure 2. Schematic representation of SENSE reconstruction with four receiver coils. The dashed lines represent missing lines in k-space. [1]

SMASH (Simultaneous acquisition of spatial harmonics)

In SMASH, the spatial dependence of the sensitivities is used to synthesize missing k-space lines by approximating the corresponding sinusoidal phase twists produced by an encoding gradient. A single k-space data set is constructed and Fourier transformed to give the final unaliased image.

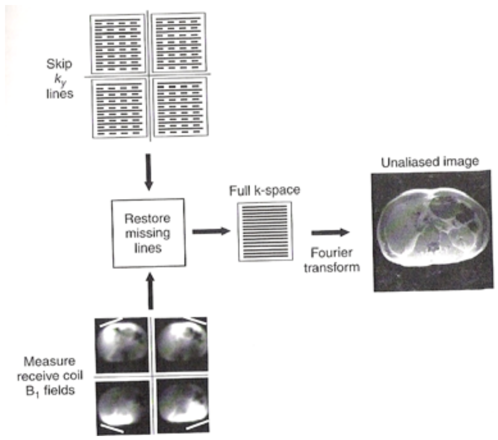


Figure 3. Schematic representation of SMASH reconstruction with four receiver coils. The dashed lines represent missing lines in k-space. [1]

SENSE

• Sensitivity encoding (SENSE), founded by *Preussmann et al.* in 1999, is a technique that enables to reduce scan time in magnetic resonance imaging (MRI) considerably.

SENSE

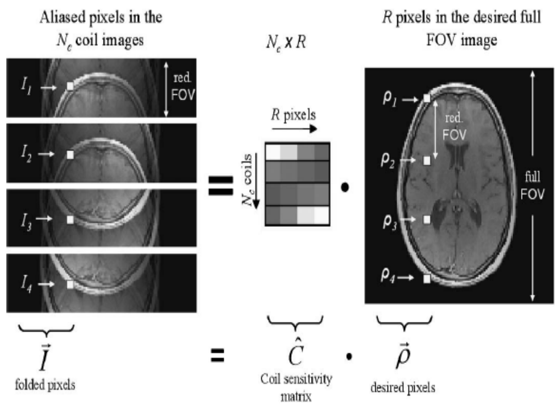


Figure 4. Illustration of the basic SENSE relation using an accelerated (R = 4) pMRI acquisition with N_c = 4 receiver coils. I contains the aliased pixels at a certain position in the reduced FOV coil images. The sensitivity matrix C assembles the corresponding sensitivity values of the component coils at the locations of the involved (R = 4) pixels in the full FOV image ρ. [5]

MATHEMATICAL EXPLANATION:

• In MRI, the reconstructed image is weighted by the component of receiver coil B_1 field that lies in the transverse plane and this fact is key to parallel imaging. For 2D imaging, $I(x,y)$ is given by,

$$I(x,y) = B_1(x,y)\, M(x,y) \qquad \ldots(2)$$

where $B_1(x,y)$ is the component of the receive B_1 field that lies in the transverse magnetization, suitably defined to include relaxation, flow, diffusion, and transmit B_1 field non-uniformity effects.

• In the rotating frame, both M and B_1 appear static for on-resonance spins. Both M and B_1 are represented by complex numbers. Introducing new subscripts that refer to each coil. Thus,

$$I_j(x,y) = C_j(x,y)\, M(x,y) \qquad \ldots(3)$$

where C is the coil B_1 field sensitivity and j refers to the j^{th} coil.

• In SENSE, the scan time is reduced by an acceleration factor of R. The FOV is therefore reduced by the same factor. If the object extends outside the reduced FOV, some pixels will be aliased or wrapped.

• This means that the image signal at an aliased pixel is a superposition of signal from a desired location in the object plus the pixels that are displaced by integer multiples of L/R, where L is the original phase encoded FOV.

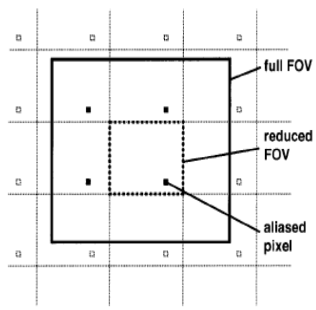


Figure 5. Aliasing in 2D Cartesian sampling: the full FOV (solid box) is reduced in both dimensions. A pixel in the reduced FOV (dotted box) represents the superposition of pixels forming a Cartesian grid. In this example four of these pixels are in the full FOV; thus the actual degree of aliasing is four. [2]

• We take phase-encoded direction as y-direction in the image. We also drop the frequency-encoded direction.

• Let the total number of replicates due to aliasing at a pixel location y be N_A. N_A is pixel dependent and is determined by R and by the size and shape of the object.

• For an object with same diameter in the phase-encoded direction as the original FOV, $N_A = R$ everywhere.

• If the object diameter is smaller than L, then some pixels will have $N_A < R$.

• If the object diameter is larger than L, some pixels will have $N_A > R$ and there will be phase wrap even without the sense of FOV reduction.

• We can express the FOV reduction mathematically by saying that a R-fold FOV reduction results in a N_A-fold aliased image representation.

• For each location y, we can write the image signal $I_j(y)$ as a superposition of original object and displaced replicates.

$$I_j(y) = \sum_{n=0}^{N_A-1} C_j(y+nL/R)\, M(y+nL/R) \qquad \text{...(4)}$$

where $j=0,1,\ldots,N_C-1$. N_C is the number of receive coils.

Depending on the location of y, the number of replicates can vary. If coil sensitivity, $C_j(y)$ can be measured, then for each value of y, then Eq. 4 would give us N_C simultaneous equations with N_A unknowns. The unknowns are aliased magnetization values $M(y+nL/R)$. $I_j(y)$ is the reconstructed aliased image.

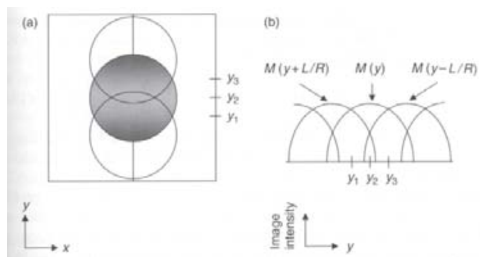


Figure 6. Aliased replicates for SENSE. (a) Image with original M(y) (shaded) and two aliased replicates, M(y+L/R) and M(y-L/R). (b) Plot of image intensity along line in (a). [1]

• If $N_C \geq N_A$, the equations can be solved to obtain $M(y+nL/R)$. Generalizing to matrix equation,

$$I = CM \qquad \text{...(5)}$$

where,

$$I = \begin{bmatrix} I_0(y) \\ I_1(y) \\ \vdots \\ I_{N_c-1}(y) \end{bmatrix}$$

$$M = \begin{bmatrix} M(y) \\ M(y+L/R) \\ \vdots \\ M(y+(N_A-1)L/R) \end{bmatrix}$$

$$C = \begin{bmatrix} C_0(y) & \cdots & C_0(y+(N_A-1)L/R) \\ \vdots & \ddots & \vdots \\ C_{N_c-1}(y) & \cdots & C_{N_c-1}(y+(N_A-1)L/R) \end{bmatrix}$$

• If $N_C \geq N_A$, the equation can be inverted to find estimated transverse magnetization *M*. The most general solution that gives maximum image SNR is pseudoinverse.

$$M = [(C^T\psi^{-1}C)^{-1}C^T\psi^{-1}]I \qquad \text{...(6)}$$

where ψ is the N_C X N_C noise correlation matrix where diagonal elements represent variance from a single coil and off-diagonal elements represent noise cross-correlation between two coils.

• If $N_C > N_A$, the inversion problem is over-determined. The extra degrees of freedom are used with noise correlation matrix to improve SNR.

• If $N_C = N_A$, the coil noise correlation matrix drops out. This is because, *C* is square and thus its inverse exists. Thus, $(C^T\psi^{-1}C)^{-1} = C^{-1}\psi(C^T)^{-1}$. We get,

$$M = [(C^T\psi^{-1}C)^{-1}C^T\psi^{-1}]I = [C^{-1}\psi(C^T)^{-1}C^T\psi^{-1}]I = C^{-1}I \qquad \text{...(7)}$$

• If in ψ the off diagonal elements are negligible and the diagonal elements are nearly equal to 1, ψ can be replaced by the Identity matrix. Thus,

$$M = [(C^TC)^{-1}C^T]I \qquad \text{...(8)}$$

• Thus for coils that are well decoupled, there is no difference in SNR results if we use Eq. 8.

• For *R=1* in SENSE, SNR is maximum. SENSE decreases the SNR as scan time is reduced. The SNR is given by,

$$SNR_{SENSE} = \frac{SNR_{NORMAL}}{g\sqrt{R}} \qquad \text{...(9)}$$

where $\sqrt{R}$ is the expected SNR loss that results from reducing scan time by a factor of *R*.

• The factor *g* is called geometric factor and represents noise magnification that occurs when aliasing is unwrapped. It is given by,

$$g_i = \sqrt{[(C^T\psi^{-1}C)^{-1}]_{ii}[(C^T\psi^{-1}C)]_{ii}} \quad ...(10)$$

where *i* refers to aliased pixel replicate number for the i^{th} pixel and has range $0,1,...,N_A-1$. Thus Eq. 10 gives geometry factor for all pixels.

• Thus, SNR is spatially dependent in SENSE.

• In general, *g* depends on the number of aliased replicates N_A and the coil sensitivity difference between aliased pixels.

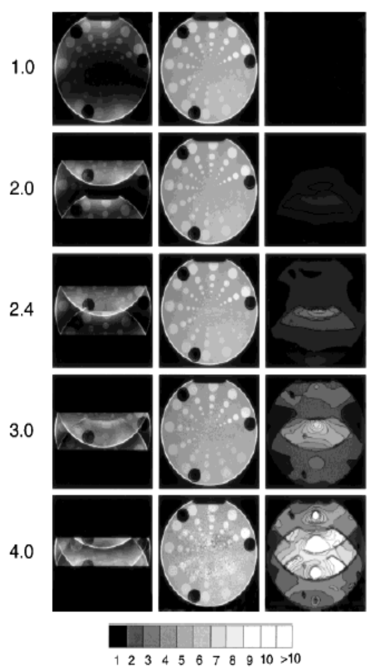
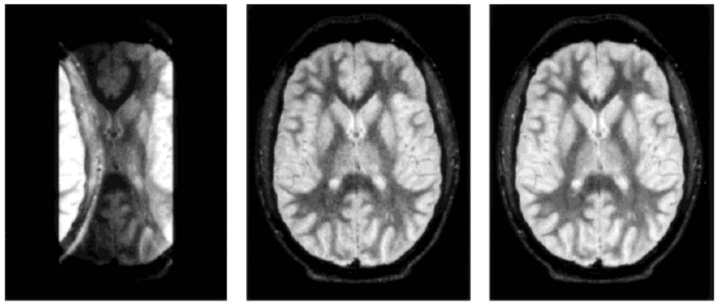


Figure 7. SENSE imaging of a quality phantom with increasing reduction factor R indicated on the left. Phase encoding in vertical direction. Left: aliased images. Middle: SENSE reconstruction from the same data. Right: maps of the relative noise level as predicted by SENSE theory, colored according to the gray-scale on the far right (arbitrary units). [2]



a b c

Figure 8. Transverse brain images obtained using two coils. (a) Reduction factor R = 2.0, conventional single-coil image and (b) SENSE reconstruction. (c) SENSE image from fully Fourier encoded data. [2]

SMASH

• SiMultaneous Acquisition of Spatial Harmonics (SMASH) is a new fast-imaging technique, developed by *Sodickson et al*. in 1997, that increases MR image acquisition speed by an integer factor over existing fast-imaging methods, without significant sacrifices in spatial resolution or signal-to-noise ratio.

SMASH

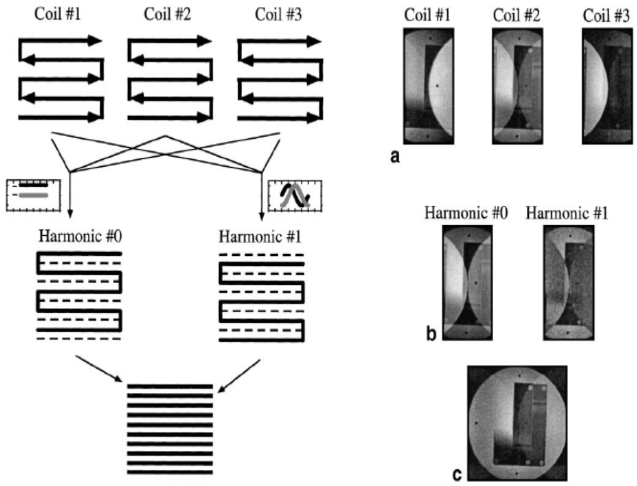


Figure 9. Schematic representation of the SMASH imaging procedure (k-space on the left, corresponding phantom images on the right). (a) Simultaneous acquisition of data with a reduced number of phase-encoding gradient steps. (b) Formation of shifted data sets using spatial harmonic combinations of component coil signals. (c) Interleaving of shifted data sets to generate a full signal matrix, corresponding to an image with full FOV in a reduced total acquisition time. [4]

SMASH Algorithm

Suppose the phase-encoding line separation is increased by a factor of *R*. Let *R* be an integer. Now SMASH uses the concept that coil sensitivities provide spatial weighting to the MR signal that is analogous to the spatial weighting provided by complex exponential functions. In SMASH, linear combinations of coil sensitivities are found that approximate to the complex exponential functions, which correspond to the omitted k-space phase-encoding lines.

Now let *y* be the phase encoded direction as per convention. Neglecting the frequency encoded direction, *x*.

For a non-parallel imaging scan, the phase encoding step is,

$$\Delta k_y = 1/L_y \quad \text{...(11)}$$

where L_y is the phase-encoding FOV.

We drop the y subscript of k and introduce m as the measured phase-encoding line number. Thus the k-space data is

$$S_j(k_m) = \int C_j(y)M(y)e^{i2\pi k_m y}dy \quad ...(12)$$

where j refers to the coil number.

Suppose $R=3$, $S_j(k_m)$ is measured for $m = 0,3,6$ and so forth. Thus, for a given k-space line m, we wish to construct three new synthetic k-space lines:

$$\hat{S}(k_m) = \int M(y)e^{i2\pi k_m y}dy \quad ...(13)$$

$$\hat{S}(k_m+\Delta k) = \int M(y)e^{i2\pi k_m y}e^{i2\pi \Delta k y}dy \quad ...(14)$$

$$\hat{S}(k_m+2\Delta k) = \int M(y)e^{i2\pi k_m y}e^{i2\pi (2\Delta k) y}dy \quad ...(15)$$

where $\hat{S}$ denotes the new k-space data.

The new k-space lines have no coil dependency and therefore no sensitivity weighting in the integrals on the right side.

The goal is to synthesize the additional complex exponentials $e^{i2\pi(p\Delta k)y}$ (where $p=0,1,2$) that appear in the integrals using coil sensitivity. If we can find the weighting factors that approximate the exponentials, the k-space can also be approximated. For example, let the coil weighting factors of $e^{i2\pi(p\Delta k)y}$ be $a_{j,p}$,

$$\sum_{j=0}^{Nc-1} a_{j,p}\, C_j(y) = e^{i2\pi(p\Delta k)y} \qquad (p=0,1,2,...R-1) \quad ...(16)$$

We can obtain new k-space data,

$$\hat{S}(k_m+p\Delta k) = \int M(y)e^{i2\pi k_m y}e^{i2\pi (p\Delta k) y}dy \quad ...(17)$$

which gives,

$$\hat{S}(k_m+p\Delta k) = \int M(y)e^{i2\pi k_m y}\sum_{j=0}^{Nc-1} a_{j,p}\, C_j(y)\, dy \quad ...(18)$$

Taking summation outside yields:

$$\hat{S}(k_m+p\Delta k) = \sum_{j=0}^{Nc-1} a_{j,p} \int C_j(y)\, M(y) e^{i2\pi k_m y}\, dy \qquad \text{...(19)}$$

$$\hat{S}(k_m+p\Delta k) = \sum_{j=0}^{Nc-1} a_{j,p}\, S_j(k_m) \qquad \text{...(20)}$$

This is the desired result. The unmeasured k-space lines without coil sensitivity weighting are constructed as a linear combination of the measured lines. New lines at the measured locations *m* are constructed the same way without coil sensitivity weighting. Coil weighting factor $a_{j,p}$ is independent of measured k-space line location k_m.

To see how to construct $a_{j,p}$, consider the following figure:

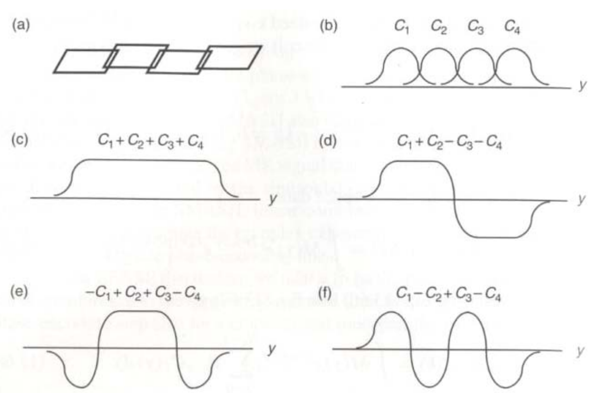


Figure 10. (a) Ladder-array of 4 coils. (b) A sample set of real part of coil sensitivities C for each coil. (c) Most multiple coils are designed such that the sum of sensitivities is constant. Therefore, the first complex exponential, $e^{i2\pi(0X\Delta k)y}=1$ can be approximated as scaled sum of exponentials. The second complex exponential $e^{i2\pi\Delta ky}=e^{i2\pi(y/L)}$ can be approximated using scaled sums and differences.
(d) Approximation of $sin(2\pi\Delta ky)$. (e) Approximation of $cos(2\pi\Delta ky)$.
(f) Approximation of sinusoidal part of third complex exponential given as $sin(2\pi(2\Delta k)y)$. [1]

This figure illustrates a problem in SMASH, that is it is sometimes difficult to approximate harmonic orders that are comparable to the number of coils in the array.

This problem can be partially solved by generalizing the SMASH formula by obtaining negative and positive harmonics.

Thus, if $R=3$, instead of obtaining $\hat{S}(k_m+2\Delta k)$ as the second positive harmonic adjacent to line k_m, we obtain $\hat{S}(k_{m+1}-\Delta k)$ as our first harmonic adjacent to line k_{m+1}. Though weighting factors for the first harmonic from k_m and k_{m+1} will be different, since a lower harmonic is being synthesized, it will be more accurate.

Another generalization is using block reconstruction. Instead of synthesizing each harmonic from a single missing line in k-space, multiple measured lines are used. If the image is divided into 4 blocks, with each having 6 missing k-space lines in each block of 8 lines, the 6 lines are synthesized from the 2 measured lines in the block. This generalization improves fitting accuracy, for coil geometries for which complex exponentials are difficult to approximate by a linear combination of coil sensitivities. The block size can be chosen to trade reconstruction speed for image quality. Larger blocks give better image quality, but slower reconstruction.

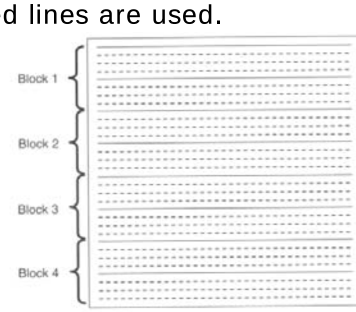


Figure 11. Illustration showing use of blocks in SMASH [1]

$$\sum_{j=0}^{Nc-1} a_{j,p}\, C_j(y) = e^{i2\pi(p\Delta k)y} \qquad (p=0,1,2,...R-1) \qquad ...(21)$$

Let,

$$f_{y,p} = e^{i2\pi(p\Delta k)y} \qquad ...(22)$$

$$c_{y,j} = C_j(y) \qquad ...(23)$$

Therefore,

$$\sum_{j=0}^{Nc-1} c_{y,j}\, a_{j,p} = f_{y,p} \qquad (p=0,1,2,...R-1) \qquad ...(24)$$

which is a matrix equation of the form,

$$ca = f \qquad ...(25)$$

where c, a and f are matrices with dimensions $N_y X N_c$, $N_c XR$ and $N_y XR$, respectively. The solution is

$$a = c^{-1} f \qquad ...(26)$$

SENSITIVITY CALIBRATION

• The image quality of parallel imaging methods is determined by how well the coil sensitivities are characterized.

• Poor characterization of the sensitivity results in uncorrected aliasing and low SNR.

• Generally, it is better to measure the sensitivity from the imaged object rather than from the coil wire geometry or with a phantom. This is because patient loading has an effect on sensitivity and also because the placement of coils cannot be predicted.

• There are mainly two methods used for measuring sensitivity:

1. Direct Measurement
2. Indirect Measurement.

Direct Measurement

Direct Measurement can be carried out in two ways. In the first method, a separate calibration scan is acquired. In the second extra data is collected during the actual scan itself for calibration. In the second method, variable density sampling is used, such that lines near the center of k-space are sampled at Nyquist frequency to reconstruct a low-resolution sensitivity map.

Although direct measurement avoids the problem of patient motion, it results in longer scan time because of fully sampled calibration lines. To avoid this, the acceleration factor is increased while sampling the outer part of k-space.

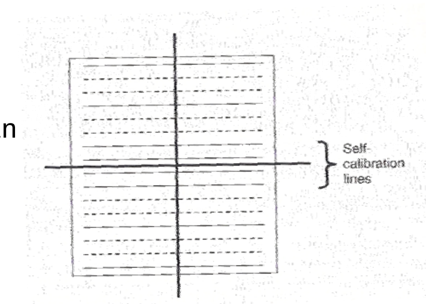


Figure 12. K-space for self calibrated parallel imaging. [1]

Indirect Measurement

In indirect sensitivity measurement, one or more extra Nyquist sampled lines are acquired near the center of k-space. This extra data is used to determine weighting factor $a_{j,p}$ for estimating missing k-space lines without explicitly computing coil sensitivity maps. The group of Nyquist sampled lines are called Auto-calibration signal (ACS) Lines.

Advanced forms of SMASH known as AUTO-SMASH and GRAPPA make use of this method for sensitivity measurement.

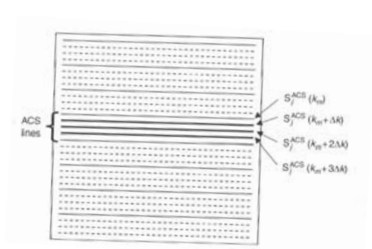


Figure 13. K-space for auto calibrated parallel imaging. [1]

Auto-Calibration SMASH (AUTO-SMASH)*

• To avoid the process of sensitivity estimation, a few extra lines are collected called the Auto-Calibration Signal (ACS) lines. These lines are used to calculate the weighting coefficients required for image reconstruction.

• For a reduction factor of R, R-1 extra lines are required for AUTO-SMASH reconstruction. This approach also helps reduce motion artifacts and reduce sensitivity to noise.

Variable Density AUTO-SMASH (VD-AUTO-SMASH)**

• For better noise insensitivity, the reconstruction coefficients for each line are calculated using the ACS and acquired lines from all coils. The process is illustrated in Fig. 14. VD-AUTO-SMASH results in acquisition of more ACS lines.

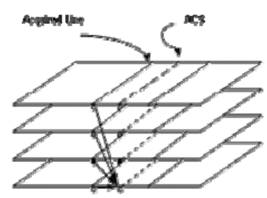


Figure 14. Use of ACS Lines in VD-AUTO-SMASH.

** P. Jacob et al., 1998.*
*** R. Heidemann et al., 2001.*

GeneRalized Autocalibrating Partially Parallel Acquisitions (GRAPPA)*

•GRAPPA further extends VD-AUTO-SMASH by considering more acquired lines per ACS line to determine the weighting factors as shown in Fig. 15. This results in further immunity to noise and also makes the reconstruction less susceptible to motion artifacts.

• In GRAPPA different acquired lines are used to generate the same ACS lines. In this case, SNR is calculated for every image and the weighting coefficients are weighted according to the SNR obtained. This approach is called the sliding block approach (Fig. 16).

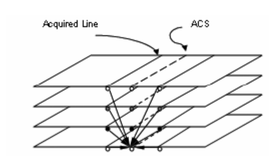


Figure 15. Use of ACS Lines in GRAPPA

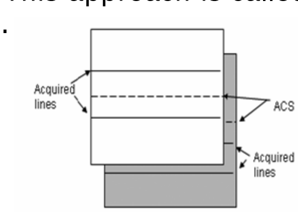


Figure 16. Sliding blocks in GRAPPA

** M. Griswold et al., 2002.*

ADVANTAGES OF PARALLEL IMAGING

- Considerable increase in imaging speed.
- High image SNR.

DISADVANTAGES OF PARALLEL IMAGING

- Although SNR is high, it decreases due to reduced scan time.

SENSE vs. GRAPPA

- Both SENSE and GRAPPA are used clinically.
- GRAPPA images have a slightly better SNR than SENSE images.
- On the other hand, GRAPPA is computationally complex and time consuming. SENSE is preferred for fast imaging.
- SENSE has better resolution while GRAPPA has better contrast.

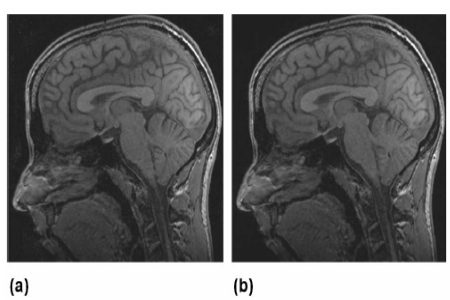


Figure 17. Comparison of the image quality of SENSE and GRAPPA reconstructions with a reduction factor of three. (a) With accurate coil sensitivity maps, the SENSE reconstruction obtains the best possible result with optimized SNR. (b) The GRAPPA reconstruction is an approximation to the SENSE reconstruction. Yet on visual scale, no differences can be seen between these methods. [5]

APPLICATIONS OF PARALLEL IMAGING

• SENSE is the most widespread used parallel MRI technique. It is used in

1. Contrast enhanced MR Angiography – SENSE enables higher spatial resolution at a constant scan time.

2. Cardiac imaging - the reduced scan time allows real-time cardiac imaging without breath-holding.

Figure 18. SENSE contrast-enhanced MRA of the abdominal aorta with R=2. [1]

• GRAPPA is also clinically used. It is used in

1. Inhomogeneous regions, such as the lungs or abdomen where it can be difficult to determine spatial coil sensitivity information.

HOMEWORK #1

• Compare SMASH and SENSE techniques used in MRI.

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