ADVANCED RECONSTRUCTION TECHNIQUES IN MRI - 2

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PARTIAL FOURIER RECONSTRUCTION

WHAT IS PARTIAL FOURIER RECONSTRUCTION?

• In Partial Fourier Reconstruction data is not collected symmetrically around the center of *k*-space.

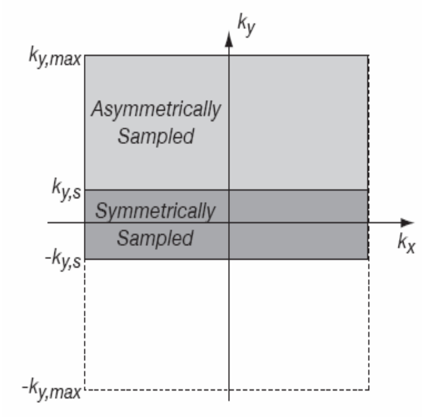
• This method takes advantage of the fact that, if the image is real, the Fourier Transform is Hermitian

$$S(-k_x, -k_y) = S^*(k_x, k_y) \qquad ...(1)$$

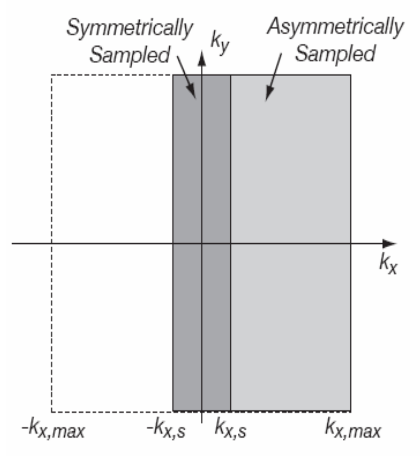
** denotes the complex conjugate*

• Thus, one half of the *k*-space is completely filled, and a small amount of additional data from the other half is collected.

PARTIAL *k*-SPACE ACQUISITION



Partial k_y



Partial k_x

SO WHY SAMPLE MORE THAN HALF THE *k*-SPACE?

- MRI is an image of weighted spin densities ρ(x) as a function of position.
- But there are unwanted phase shifts resulting from motion, resonance frequency offsets, hardware delays, eddy currents, and B1 field inhomogeneity.
- These cause the signal to be complex instead of being purely real.
- The data collected in the incompletely filled half of *k*-space shown is used to overcome the problem.

ZERO FILLING

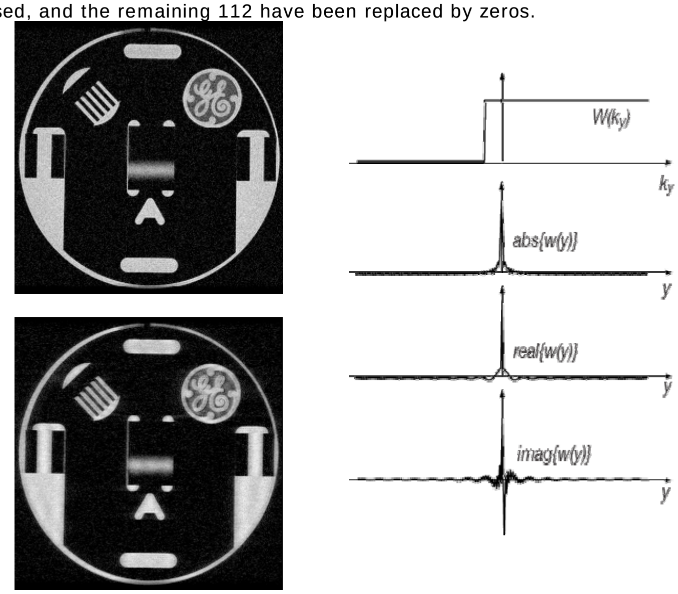
• The simplest way to reconstruct a partial k-space data set is to simply fill the uncollected data with zeroes.

• After filling with zeroes, we perform the standard 2D Fourier Transform and display the magnitude.

• This results in Gibbs ringing near the sharp edges due to truncation of k-space data.

• This works acceptably if the collected k-space fraction is close to 1, and works poorly as this fraction approaches 0.5. Usually, a reasonable phase accuracy is achieved by acquiring a relatively high fraction of k-space, greater than 0.75.

Fig. 1 Comparison of a reconstruction of a full k-space data, and a trivial partial k-space reconstruction of the same data set where only 144 of 256 phase encodes have been used, and the remaining 112 have been replaced by zeros.



HOMODYNE PROCESSING*

• Homodyne processing uses low spatial frequency phase map generated from the data itself to correct for phase errors produced by the reconstruction of incomplete k-space data.

• Homodyne processing exploits the Hermitian conjugate symmetry of k-space data that would result if the reconstructed object were real.

** Noll et al., 1991*

Mathematical Explanation

Consider a 1-D case. Let the k-space data $S(k)$ for full Fourier acquisition extend from $-k_{max}$ to $+k_{max}$. Suppose, in the partial Fourier acquisition, k-space data is acquired from $-k_0$ to $+k_{max}$ where k_0 is positive. It can be assumed that k-space data is sampled symmetrically about the low frequency region between $(-k_0, +k_0)$ around $k=0$ and sampled unsymmetrically about the high frequency region between (k_0, k_{max}).

Now, the algorithm is divided into 2 steps:

1. Hermitian Conjugate Replacement of missing data
2. Correction for the Imaginary Component

Hermitian Conjugate Replacement:

Assuming an ideal case – for symmetrically sampled k-space data, the reconstructed image is real and is given by,

$$I(x) = \int_{-k_{max}}^{+k_{max}} S(k)\, e^{j2\pi kx}\, dk \qquad \text{...(2)}$$

Data in the range $(-k_{max},-k_0)$ can be replaced by complex conjugate of data in the range (k_0,k_{max}), resulting in,

$$I(x) = \int_{-k_{max}}^{-k0} S^*(-k)\, e^{j2\pi kx}\, dk + \int_{-k0}^{+k_{max}} S(k)\, e^{j2\pi kx}\, dk \qquad \text{...(3)}$$

In the first term, let $k'=-k$, resulting in,

$$I(x)= \int_{-k_{max}}^{-k0} S^*(k')e^{-j2\pi k'x}(-dk') + \int_{-k0}^{+k_{max}} S(k)\, e^{j2\pi kx}\, dk \qquad \text{...(4)}$$

Thus,

$$I(x) = \left[\int_{k0}^{k_{max}} S(k')\, e^{j2\pi k'x}\, dk'\right]^* + \int_{-k0}^{+k_{max}} S(k)\, e^{j2\pi kx}\, dk \qquad \text{...(5)}$$

The second term can be split into two, between $(-k_0,k_0)$ and (k_0,k_{max}). Combining the second of the two terms with the first term, gives,

$$I(x) = \int_{-k0}^{+k0} S(k)\, e^{j2\pi kx}\, dk + 2\, Re\left[\int_{k0}^{k_{max}} S(k)\, e^{j2\pi kx}\, dk\right] \qquad \text{...(6)}$$

Since, the sum of a complex number and its conjugate is equal to twice the real part. Thus, the second term is real by definition, and since we assume that final image *I(x)* is real, first term must also be real. Therefore,

$$I(x) = Re[\int_{-k0}^{+k0} S(k)\, e^{j2\pi kx}\, dk + 2 \int_{k0}^{kmax} S(k)\, e^{j2\pi kx}\, dk] \quad ...(7)$$

Which can be further simplified by defining a function H(k) given as,

$$H(k) = \begin{cases} 0 & k < -k_0 \\ 1 & -k_0 \le k < k_0 \\ 2 & k \ge -k_0 \end{cases} \quad ...(8)$$

to give, $I(x) = Re[I_H(x)]$ ...(9)

where, $I_H(x) = \int_{-kmax}^{+kmax} H(k)\, S(k)\, e^{j2\pi kx}\, dk$ ...(10)

The function *H(k)* is called the Homodyne High Pass Filter.

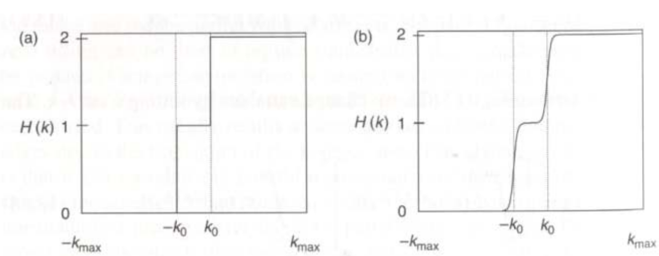


Fig. 2 Homodyne high-pass filter. (a) Conceptual version (b) Apodized version with smooth transitions.

• Eq. 9 and Eq. 10 implies that instead of using the Hermitian conjugate symmetry for reconstruction, the partial Fourier data can be reconstructed using Homodyne High-pass filter.

• This method zero fills the missing data and doubles the weighting of the asymmetrically sampled data.

• The operation of taking the real part is required because doubling the asymmetrically sampled high frequencies generates an unwanted imaginary component.

•Abrupt transitions in H(k) cause ringing.

•The transition smoothing of the above filter could be obtained by using the appropriate window. The resulting Homodyne HPF would be:

$$H(k) = \begin{cases} 0 & k \le -k_0 - w/2 \\ \cos^2\left(\dfrac{\pi(|k| - (k_0 - w/2))}{2w}\right) & -k_0 - w/2 < k < -k_0 + w/2 \\ 1 & -k_0 + w/2 \le k \le k_0 - w/2 \\ 1 + \cos^2\left(\dfrac{\pi(|k| - (k_0 + w/2))}{2w}\right) & k_0 - w/2 < k < k_0 + w/2 \\ 2 & k \ge k_0 + w/2 \end{cases} \quad \text{...(11)}$$

Correction for the Imaginary Component:

- Since $I(x)$ is not purely real, the operation of taking the real part discards some desired signal.

- This problem can be avoided by phase correction.

- In Homodyne filtering, the phase correction is derived from the symmetrically sampled k-space data.

- A low frequency image $I_L(x)$ is reconstructed from:

$$I_L(x) = \int_{-k0}^{+k0} S(k)\, e^{j2\pi kx}\, dk = \int_{-kmax}^{kmax} L(k)\, S(k)\, e^{j2\pi kx}\, dk \quad ...(12)$$

where: $$L(k) = \begin{cases} 1 & |k| \le k_0 \\ 0 & |k| > k_0 \end{cases} \quad ...(13)$$

$L(k)$ is a low pass filter.

In practice, a filter with smooth transition is used.

$$L(k) = \begin{cases} 1 & |k| \le k_0 - w/2 \\ \cos^2\left(\dfrac{\pi(|k| - (k_0 - w/2))}{2w}\right) & k_0 - w/2 < |k| < k_0 + w/2 \\ 0 & |k| \ge k_0 + w/2 \end{cases} \quad ...(14)$$

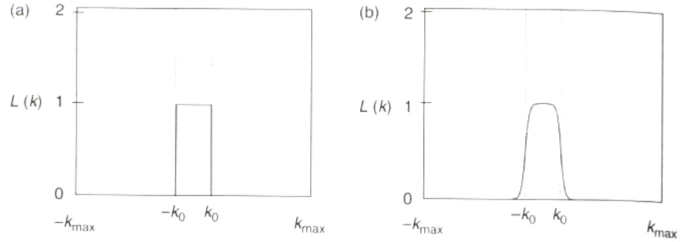


Fig. 3 Homodyne low-pass filter. (a) Conceptual version (b) Apodized version with smooth transitions.

• We approximate the phase of $I(x)$ to the phase of $I_L(x)$, denoted by $\Phi_L(x)$.

• Thus, the phase corrected image is denoted by $I_H(x)e^{-\Phi_L(x)}$ has $I(x)$ registered to the real part of the image, allowing the undesired imaginary component from the homodyne filter.

• Thus, the entire homodyne reconstruction can be expressed as

$$I(x) = Re[I_H(x)e^{-\Phi_L(x)}] \quad \text{...(15)}$$

• To avoid phase wrapping from using an arctangent function, it is preferable to avoid calculating $\Phi_L(x)$ explicitly. Instead, we evaluate Eq. 15 using,

$$I_H(x)e^{-\Phi_L(x)} = I_H(x)\frac{I_L^*(x)}{|I_L(x)|} \quad \text{...(16)}$$

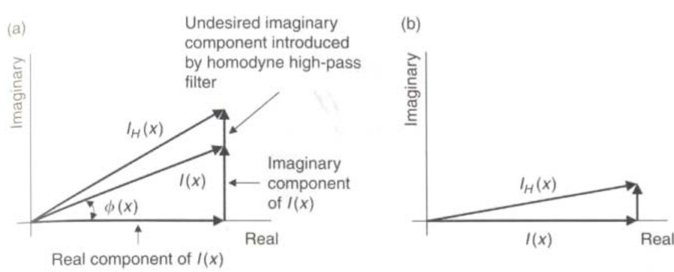


Fig. 4 Complex image value at one pixel (a) before phase correction and (b) after phase correction. $I(x)$ has phase $\Phi(x)$ which we approximate as $\Phi_L(x)$ the phase of low-pass filtered image. The phase correction removes $\Phi_L(x)$, thereby approximately registering I(x) into real part of the phase corrected image $I_H(x)e^{-i\Phi_L(x)}$. This allows discarding the imaginary image component introduced by homodyne high-pass filter.

• The approximation that the phase of the image can be represented by $\Phi_L(x)$ means that homodyne reconstruction performs relatively poorly in region of rapidly varying phase caused by susceptibility changes.

• Iterative methods provide improved performance.

EXTENSION TO 2-D AND 3-D SPACE

• Earlier we considered a 1-D k-space for simplicity. Expanding to the 2-D domain

$$S(-k_x,-k_y) = S^*(k_x,k_y) \qquad \text{...(17)}$$

• Suppose that the partial Fourier acquisition were used in both the k_x and k_y directions with partial Fourier fraction of 0.5, thus acquiring only one of the four quadrants of 2D k-space.

• Using Eq. 16 helps us fill only the diagonally opposed quadrant, leaving two other quadrants empty.

• Thus if partial Fourier is used in two orthogonal directions, one direction can be processed with homodyne reconstruction but the second direction must use zero filling.

• Conversely, if partial Fourier is used in one direction, the other k-space directions must be processed first with the normal algorithm.

• Consider a 2-D case in which full Fourier acquisition is used in k_x direction whereas partial Fourier is used in the k_y direction.

• Taking 1-D F.T. of the fully sampled k_x direction results in partially transformed data $S_p(x,k_y)$ given by,

$$S_p(x,k_y) = \int S(k_x,k_y)\, e^{j2\pi k_x x}\, dk_x \qquad ...(18)$$

where $S_p(x,k_y)$ is called the signal in hybrid space.

• The Hermitian conjugate of $S_p(x,k_y)$ with respect to k_y is,

$$S_p^*(x,-k_y) = \int S^*(k_x,-k_y)\, e^{-j2\pi k_x x}\, dk_x \qquad ...(19)$$

• Using Hermitian relationship $S^*(k_x,-k_y)=S(-k_x,k_y)$ yields,

$$S_p^*(x,-k_y) = \int S(-k_x,k_y)\, e^{-j2\pi k_x x}\, dk_x \qquad ...(20)$$

• Finally, $S_p^*(x,-k_y)=S_p(x,k_y)$ ...(21)

• Because the partially transformed data obeys the same Hermitian relationship as 1D k-space data, the 2D data is first processed normally in the full Fourier direction (k_x), followed by partial processing in the k_y direction.

• The extension to three dimensions is straightforward.

Fig 5. The flowchart for the same is shown below.

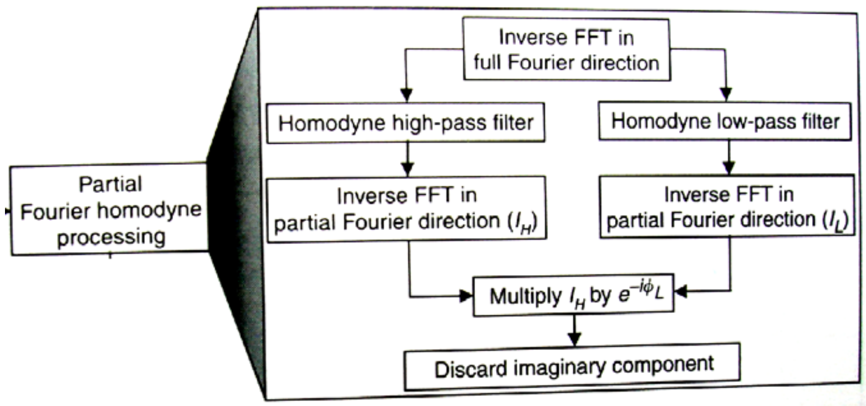
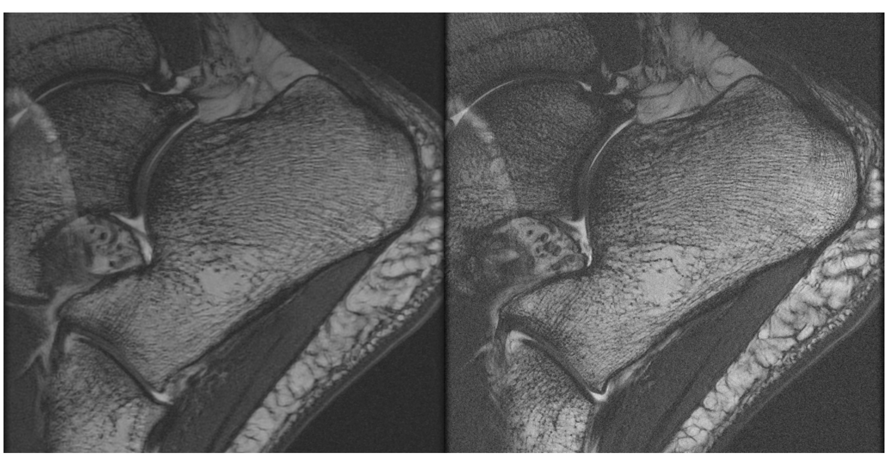


Fig. 6 Comparison between zero-filled and homodyne processing:

Zero-filled

Homodyne Processing



DISADVANTAGES OF HOMODYNE RECONSTRUCTION

• As seen in Eq. 16, image phase is lost with homodyne reconstruction. Hence it is unsuitable for applications such as shimming, phase contrast, and phase sensitive thermal imaging.

ITERATIVE HOMODYNE PROCESSING

• A drawback of the Homodyne method is that the low-frequency phase map used cannot accurately determine the rapidly varying phase.

• To address this problem, iterative partial Fourier Transform has been developed.

• In this method, an image is formed using Homodyne Reconstruction.

• This image is then Fourier transformed to obtain estimated k-space data.

• The original k-space data in the range $(-k_0, k_{max})$ are combined with the newly estimated k-space data in the range $(-k_{max}, -k_0)$ and a new complex image I' is calculated.

• A new magnitude image is formed by applying the low-frequency phase correction to I' and taking the real component, as in non-iterative homodyne reconstruction.

• This magnitude image is input to the next iteration.

Mathematical Description

• Consider 1-D case. For the first iteration, $I_0(x)$ is given by

$$I(x) = Re[I_H(x)e^{-\Phi_L(x)}] \quad \text{...(21)}$$

• Let $S_j(k)$ be the complex k-space data estimated at step j. Iteration starts by computing $S_0(k)$. At each step:

$$S_j(k) = FT[I_j(x)e^{i\Phi_L(x)}] \quad \text{...(22)}$$

• This function is an estimate of the k-space value for all values of k.

• However in the range $(-k_0, k_{max})$, $S(k)$ is more accurate.

• Hence we use $S(k)$ in the range $(-k_0, k_{max})$ and $S_j(k)$ in the range $(-k_{max}, -k_0)$ should yield a better-estimated image for the next iteration step.

• Combining the two data sets, creates a discontinuity at $k=-k_0$. Therefore, it is better to smoothly blend the two datasets to obtain the estimated k-space data,

$$S_{j+1} = W(k)\,S(k) + [1 - W(k)]\,S_j(k) \qquad \text{...(23)}$$

where W(k) is a merging function,

$$W(k) = \begin{cases} 0 & k \le -k_0 - w_m/2 \\ \cos^2\left(\dfrac{\pi(|k| - (k_0 - w_m/2))}{2w_m}\right) & -k_0 - w_m/2 < k < -k_0 + w_m/2 \\ 1 & k \ge -k_0 + w_m/2 \end{cases} \qquad \text{...(24)}$$

where w_m is the merging width.

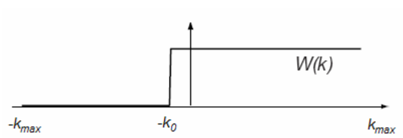


Fig. 7 Merging function W(k) for iterative homodyne reconstruction

- The image for the next iteration $I_{j+1}(x)$ is:

$$I_{j+1}(x) = Re\{e^{-i\Phi_L(x)}\,FT^{-1}[S_{j+1}(k)]\} \qquad \text{...(25)}$$

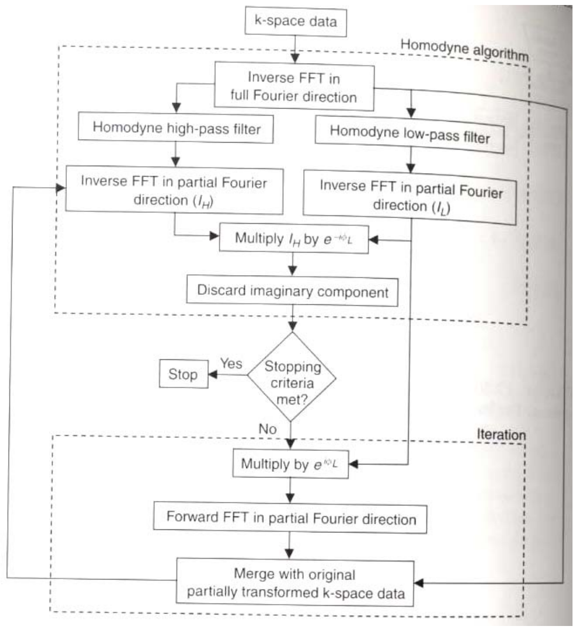
- Many choices for stopping criteria are possible.

  1. The algorithm continues for a fixed number of iterations.

  2. It could continue until the difference between two successive iterations becomes sufficiently small.

- Extension to 2D and 3D is similar to a non-iterative homodyne algorithm – normal reconstruction is performed first in the fully sampled Fourier directions.

Fig. 8 The flowchart for the same is shown below.



HOMEWORK #2

For the given k-space data set, perform the following:

1) Consider 60% of the given k-space data-set. Reconstruct the k-space using zero filing. Plot the final image thus obtained.

2) Consider 50% of the given k-space data-set. Reconstruct the k-space using Hermitian Conjugate Symmetry. Plot the final image thus obtained.

3) Consider the k-space dataset given. Generate a new k-space by taking only every 4th line from the original k-space. Plot the resultant image. What can you see?

REFERENCES

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