Advanced MRI
BioE 594

REVIEW

Lecture outline

- Signal detection
- Signal and Image Processing for MRI
- Imaging Gradients
- Correction Gradients

Signal detection

Read:
•E.L. Hahn. Spin Echoes. Phys. Rev., 80:580, 1950.

Signal Reception in MRI

- MR signal: voltage induced in the RF coil by changes in magnetic flux from the precessing magnetization in our sample.

$$emf = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

where Φ is the flux in the coil.

Signal Equation

$$s(t) = \int_x \int_y m(x,y) e^{-i2\pi[k_x(t)x + k_y(t)y]} dxdy$$

where

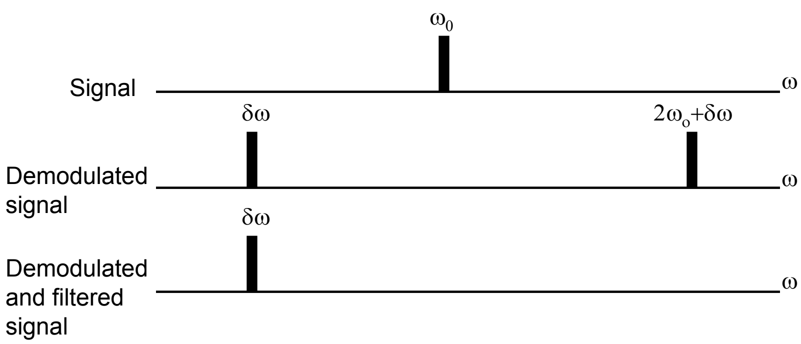
$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau$$

Signal Demodulation

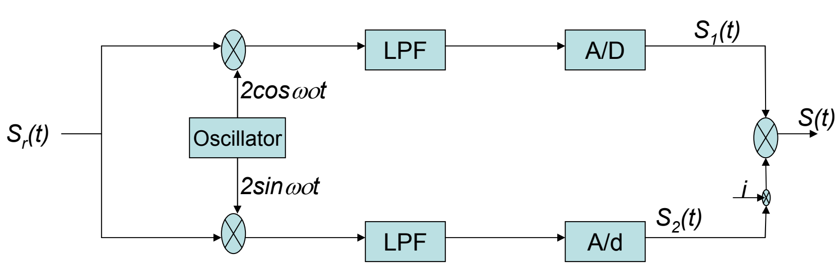
- Demodulation corresponds to the multiplication of the signal by a sinusoid or cosinusoid with a frequency near or at $\omega_{0.}$
- The high-frequency (MHz) is demodulated in a part of the receiver (demodulator) and converted to a low frequency (kHz) signal that contains the "modulated information", i.e. the frequency range across the field of view encoded by the frequency encoding gradient.
- Both sine and cosine multiplication are considered corresponding to data storage in two channels respectively, real and imaginary.
- Low pass filtering applied to the demodulated signal eliminates the high frequency components.

Demodulated Signal



The shift in frequency for signals from a uniform sample due to demodulation and filtering

Signal Demodulation



Signal Demodulation

The upper channel

$$s_1(t) = LPF\{2m_0 \cos(\omega_0 t + \Delta\omega t)\cos(\omega_0 t)\}$$

$$= LPF\{m_0[\cos(\Delta\omega t) + \cos(2\omega_0 t + \Delta\omega t)]\}$$

$$= m_0 \cos(\Delta\omega t)$$

The lower channel

$$s_2(t) = LPF\{2m_0 \cos(\omega_0 t + \Delta\omega t)\sin(\omega_0 t)\}$$

The negative has been chosen as a convention

$$= LPF\{m_0[-\sin(\Delta\omega t) + \sin(2\omega_0 t + \Delta\omega t)]\}$$

$$= -m_0 \sin(\Delta\omega t)$$

The combined signal

$$s(t) = s_1(t) + is_2(t)$$

$$= m_0 \exp(-i\Delta\omega t)$$

$$= m_0 \exp(-i\Delta\omega t)$$

$$\sin s + \sin t = 2 \sin \frac{s+t}{2} \cos \frac{s-t}{2}$$

$$\sin s - \sin t = 2 \cos \frac{s+t}{2} \sin \frac{s-t}{2}$$

$$\cos s + \cos t = 2 \cos \frac{s+t}{2} \cos \frac{s-t}{2}$$

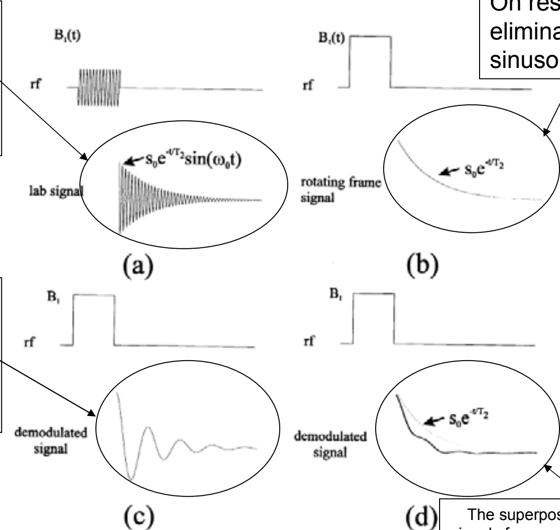
$$\cos s - \cos t = -2 \sin \frac{s+t}{2} \sin \frac{s-t}{2}$$

Free Induction Decay and T_2^*

Demodulation On resonance eliminates the sinusoid with only T2

No Demodulation is applied, rapid oscillations at frequency ω are damped by T2

Demodulation Slightly off resonance leaves a low frequency components



The superposition of demodulated signals from several spins populations, each with different ω_0, leads to a signal that decay that decays faster than a single demodulated signal which only has T2 damping

- FID: free – meaning it is not being driven by an RF pulse, induction – the action of a magnetic moment precessing around a magnetic field, and decay – meaning T2 decay).

Variations of the Magnetic Field

- There many things that can affect the magnetic field. These include
  - Magnetic field inhomogeneity – this reflects our inability to make the field perfectly homogeneous.
  - Magnetic susceptibility – this is the magnetization of tissue itself. Different tissues, bones and the surrounding air all have magnetic susceptibility differences of several ppm. The net field is given as $B = B0(1+\chi)$, where χ is the magnetic susceptibility (χ_{air} is nearly 0, χ_{water} is about –9x10-6 or –9 ppm).

Variations of the Magnetic Field (cont'd)

– Chemical shift –different shielding of the nucleus from the surrounding electron clouds. The net field is $B = B0(1-\sigma)$, where σ is the chemical shift (a positive chemical shift implies shielding of the nucleus or a downward shift in the field). A common chemical shift the shift between water protons (bonded to O) and fat protons (bonded to C): σ_{wf} is about 3.5 ppm.

Signal and Image Processing for MRI

Fourier Transformation

- The Fourier transform (FT) of a time-domain function g(t) is a frequency-domain function G(ν), or spectrum

$$G(\nu) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi\nu t} dt$$

•Extract from g(t) the amplitude of the frequency component at frequency ν.
•The inverse FT (IFT) describes the synthesis of a time domain signal from sinusoidal components:

$$g(t) = \int_{-\infty}^{\infty} G(\nu) e^{j2\pi\nu t} d\nu$$

time, t, and frequency, ν, form a FT pair

Fourier Transformation

- In MRI the FT pair: spatial position vectors, **x** = (x, y), and spatial frequency vectors, **k** = (k_x, k_y)
- What is the unit of spatial frequency?

$$G(k_x, k_y) = \iint g(x, y) e^{-j2\pi(k_x x + k_y y)} dxdy$$

$$g(x, y) = \iint G(k_x, k_y) e^{j2\pi(k_x x + k_y y)} dk_x dk_y$$

Sufficient conditions for *g(x,y)*
•Continuous
•integrable

Properties of FT

- Linear: $\mathrm{FT}[a_1 g_1(x,y) + a_2 g_2(x,y)] = a_1 \mathrm{FT}[g_1(x,y)] + a_2 \mathrm{FT}[g_2(x,y)]$
- Translation $\mathrm{FT}[g(x - x_0, y - y_0)] = \mathrm{G}[u,v] e^{-i(ux_0 + vy_0)}$
- Scale $\mathrm{FT}[g(|a|x, |b|y)] = \frac{1}{|ab|} \mathrm{G}[\frac{u}{|a|}, \frac{v}{|b|}]$

Convolution

$$\mathrm{FT}[f(x,y)] = \mathrm{F}[u,v]$$
$$\mathrm{FT}[g(x,y)] = \mathrm{G}[u,v]$$
then
$$\mathrm{FT}[\iint f(x,y) g(x'-x, y'-y) dxdy] = F(u,v)G(u,v)$$

Dirac delta function

$$\delta(x) = \begin{Bmatrix} \infty & x = 0 \\ 0 & x \neq 0 \end{Bmatrix}$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \qquad unit \qquad response$$

$2D$

$$\delta(x, y) = \delta(x)\delta(y)$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x)\delta(y) dxdy = 1$$

Sifting

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\delta(x - x_0)\delta(y - y_0) dxdy = f(x_0, y_0)$$

FT(δ) = δ

k-Space

$$s(t) = \iint m_{xy,rot}(x, y, t) dxdy$$

Consider a spatially and temporally varying applied magnetic fields introduced by time varying gradient fields:

$$B(x, y, t) = B_0 + G_x(t) \cdot x + G_y(t) \cdot y$$

Then for a rotating frame

$$\Delta\omega(x, y, t) = \gamma\left(G_x(t) \cdot x + G_y(t) \cdot y\right)$$

and

$$\phi(x, y, t) = \int_0^t \gamma\left(G_x(\tau) \cdot x + G_y(\tau) \cdot y\right) d\tau$$

K-Space (cont'd)
Signal equation

$$s(t) = \iint m_{xy,rot}(x,y,t)dxdy$$
$$= \iint m(x,y)\exp(-i\phi(x,y,t))dxdy$$
$$= \iint m(x,y)\exp\left(-i\int_0^t \gamma\big(G_x(\tau)\cdot x + G_y(\tau)\cdot y\big)d\tau\right)dxdy$$
$$= \iint m(x,y)\exp\left(-i\gamma\left(\int_0^t G_x(\tau)d\tau\cdot x + \int_0^t G_y(\tau)d\tau\cdot y\right)\right)dxdy$$

with

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

$$s(t) = \iint m(x,y)\exp\big(-i2\pi\big(xk_x(t)+yk_y(t)\big)\big)dxdy$$
$$= F_{2D}\{m(x,y)\}|_{u=k_x(t),v=k_y(t)} = M\big(k_x(t),k_y(t)\big)$$

$$s(t) = \mathrm{FT}\{m(x,y)\} = M\big(k_x(t),k_y(t)\big)$$

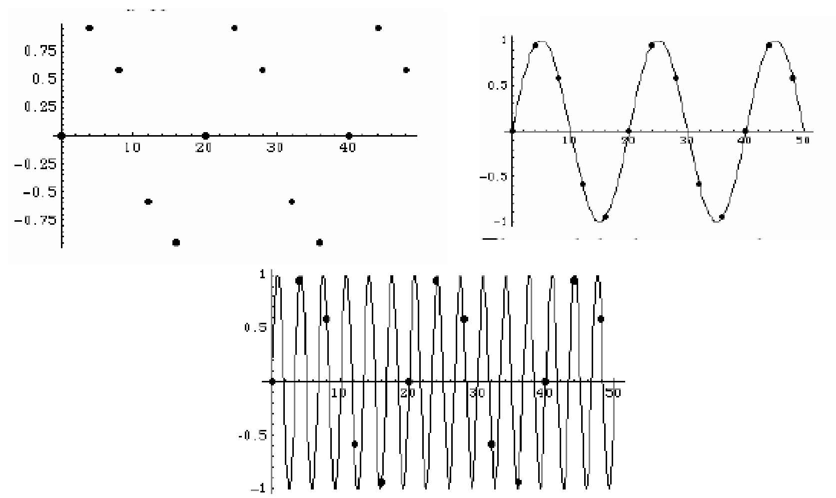
→ the signal is equal to the Fourier transform of the initial magnetization evaluated at locations defined by the *k-space*

- So
  - What is k space
  - Where does it start
  - What control the k-space trajectory
  - Ok we sample FT of the magnetization, how do we get the image

Sampling in *k* space and Aliasing

- Aliasing: An aliased frequency is a high frequency temporal or spatial signal component that is represented at an low frequency. This results from sampling at too low a rate to faithfully capture high frequency components.

Aliasing



Nyquist criterion

- For a bandlimited time-domain signal with highest frequency component ν_{max}, aliasing will not occur if the sampling rate, ν_s, satisfies $\nu_s > 2\nu_{max}$

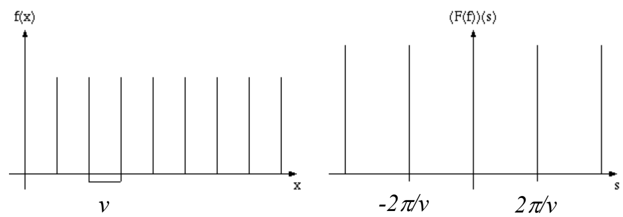
DFT

$$G(\tfrac{n}{N\Delta T}) = \sum_{m=0}^{N-1} g\left(m\Delta T\right) e^{-j2\pi nm/N}$$

Digital convolution of two data streams of length N, x1[n] and x2[n], is defined by

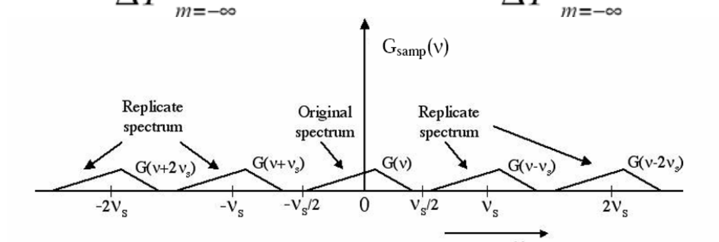
$$c[n] = x_1[n] * x_2[n] \equiv \sum_{k=0}^{N-1} x_1[k]\, x_2[n-k]$$

Comb Functions: infinite series of equidistant Dirac impulses, where adjacent impulses are a distant of v apart



$$g_{samp}(t) = g(t) \bullet \sum_{n=-\infty}^{+\infty} \delta(t - n\Delta T) = \frac{1}{\Delta T} \sum_{m=-\infty}^{m=+\infty} g(t) e^{j2\pi m v \Delta T}$$

$$G_{samp}(\nu) = \frac{1}{\Delta T} \sum_{m=-\infty}^{m=+\infty} G(\nu - m/\Delta T) = \frac{1}{\Delta T} \sum_{m=-\infty}^{m=+\infty} G(\nu - m\nu_s)$$



The original spectrum is preserved without distortion as long as the frequency spectrum G(v) is bandlimited to frequencies |v| < vs/2 and this is the Nyquist criterion

Sampling in MRI

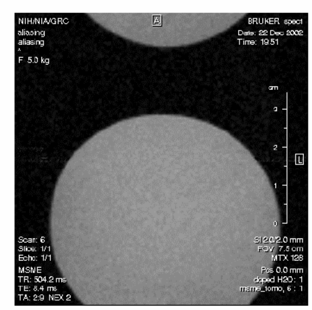
- Sampling of *k* space must be of high enough frequency to properly represent high-frequency spatial components.
- Consider the read direction

$$\gamma G_r FOV_r = \text{Sampling Bandwidth} = 1/\Delta T$$

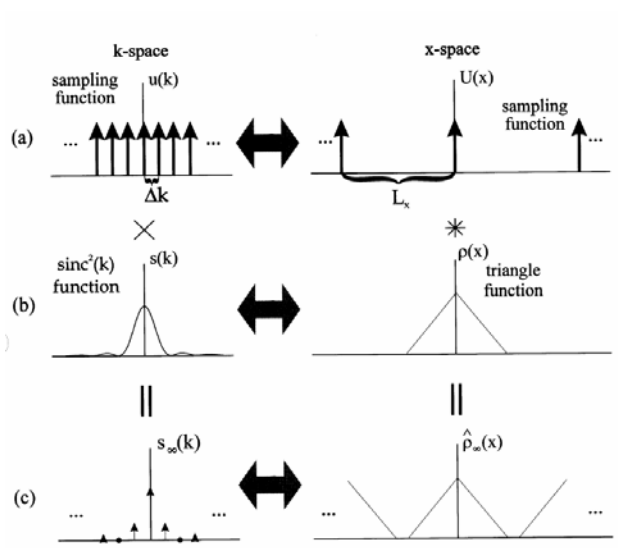
but $\Delta k_r = \gamma G_r \Delta T$

$$\Delta T < 1/(\gamma G_r W_r)$$

$$\Rightarrow \quad FOV_r = 1/\Delta k_r$$



Sampling concept



Sampling MR k space

$$\widetilde{M}(u,v) = M(u,v)\,\mathrm{comb}\left(\frac{u}{\Delta k_x}, \frac{v}{\Delta k_y}\right)$$

$$= \Delta k_x \Delta k_y \sum_{n,m=-\infty}^{\infty} \delta(u - n\Delta k_x, v - m\Delta k_y) M(n\Delta k_x, m\Delta k_y)$$

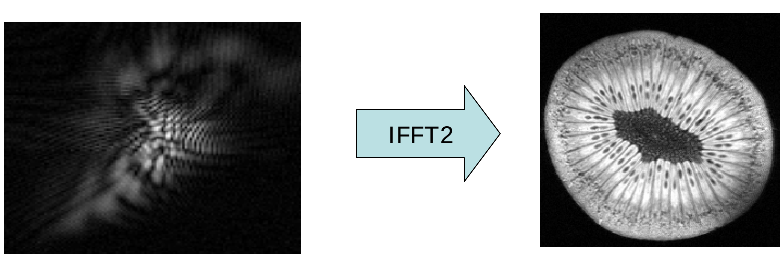
The image (space) domain equivalent is:

$$\widetilde{m}(x,y) = m(x,y) ** \Delta k_x \Delta k_y \mathrm{comb}(\Delta k_x u, \Delta k_y v)$$

$$= m(u,v) ** \sum_{n,m=-\infty}^{\infty} \delta\left(u - \frac{n}{\Delta k_x}, v - \frac{m}{\Delta k_y}\right)$$

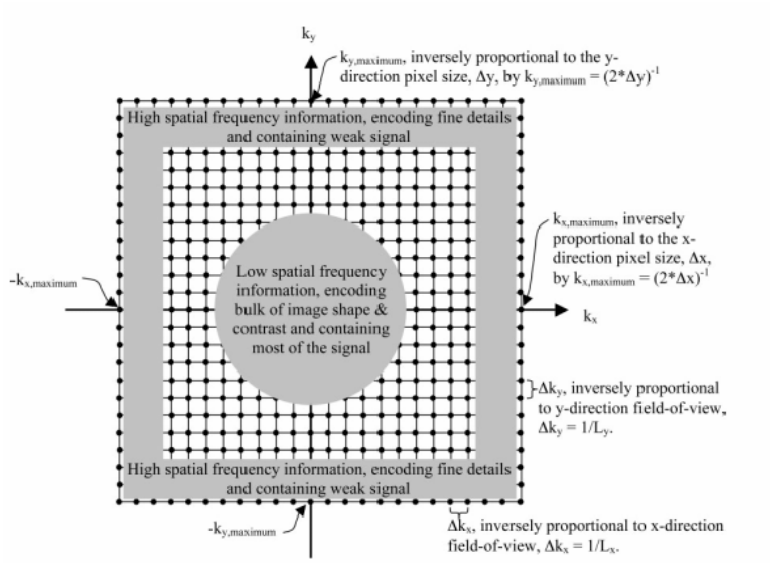
$$= \sum_{n,m=-\infty}^{\infty} m\left(u - \frac{n}{\Delta k_x}, v - \frac{m}{\Delta k_y}\right)$$

$\mathrm{FOV}_x = 1/\Delta k_x$ and $\mathrm{FOV}_y = 1/\Delta k_y$



k space *image* space

K space



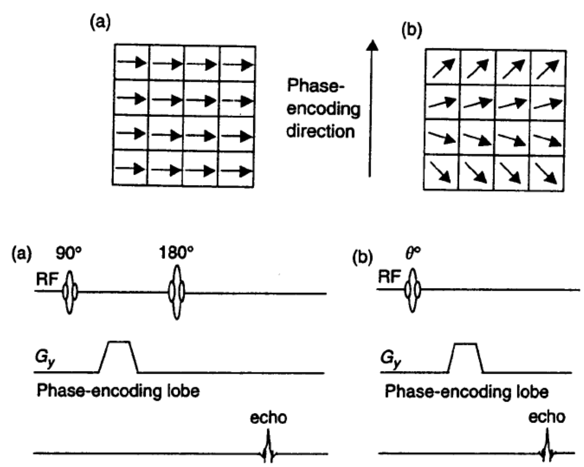
Imaging Gradients

Reference: Handbook of pulse sequences

Phase-Encoding Gradients

- Spatial Localization in MRI employs both phase and frequency encoding.
- Phase encoding creates a linear spatial variation of the phase of the magnetization.
- Phase : angle made by the transverse magnetization vector with respect to some fixed axis in the transverse plane

Phase Encoding



Phase-Encoding

- Phase encoding must be applied before readout gradient
- Different phase variation is introduced by changing the area under the phase encoding gradient
- Phase encoding is used to spatially encode information orthogonal to the frequency-encoded direction

Mathematical description

- For a y –phase encoding gradients G_y

$$\omega = \gamma G_y y$$

$$\phi = \gamma y \int_0^T G_y(\tau) d\tau = 2\pi k_y y \qquad (1)$$

- The effective magnetization M_p=M_x+iM_y

$$S(k_y) = \int M_p(y) e^{-i\phi(y)} dy \qquad (2)$$

Phase-Encoding (cont'd)

- Transforming (2) to a discrete eq. using (1)

$$S(k_y) = \sum_{n=0}^{N-1} M_p(n\Delta y) e^{-2\pi i (n\Delta y) k_y} \qquad (3)$$

- Repeat the phase encoding steps for N times
- For N phase encoding lines, the area covered in k-space is $(N-1)\Delta k_y$

Phase-Encoding (cont'd)

- For N phase-encoding step acquired sequentially starting at the top edge of k-space

$$k_y(m) = k_{y,\max} - m\Delta ky \qquad m = 0,1,\ldots,N-1$$

$$k_{y,\max} = \frac{1}{2}(N-1)\Delta ky$$

$$\Rightarrow k_y(m) = (\frac{N-1}{2} - m)\Delta ky \qquad (4)$$

The signal:

$$S(m) = \sum_{n=0}^{N-1} M_p(n\Delta y) e^{-2\pi i (n\Delta y)(\frac{N-1}{2} - m)\Delta ky} \qquad m = 0,\ldots,N-1 \qquad (5)$$

Phase-Encoding (cont'd)

Δky chosed based on the Nyquist criteerion

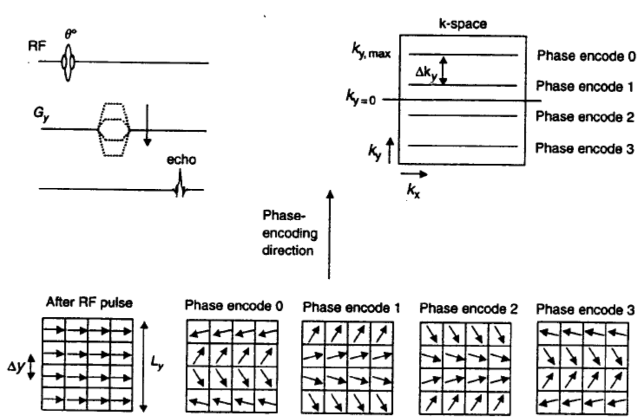
$$\Delta ky = \frac{1}{FOV_y} = \frac{1}{N\Delta y}$$

$$N\Delta k_y = \frac{1}{\Delta y}$$

equation 5 becomes

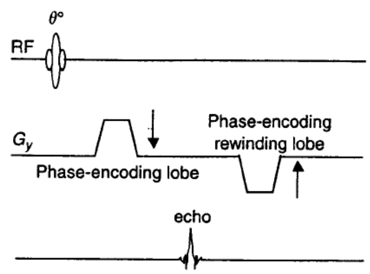
$$S(m) = \sum_{n=0}^{N-1} M_p(n\Delta y) e^{-\pi in(N-1)/N} e^{-2\pi imn/N} \qquad (6)$$

Gradient echo with four phase encoding steps



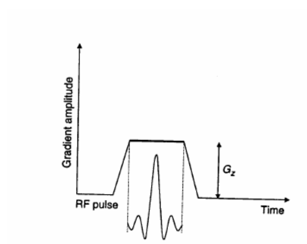
Rephasing Lobe

- For each phase encoding step a rephaser with a negative area is applied



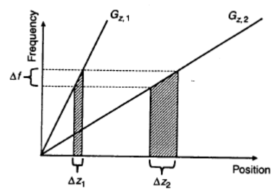
Slice Selection Gradients

- Spatially selective RF pulses require a slice-selection gradient.
- The slice selection gradient is a constant gradient that is played concurrently with the selective RF pulse.



Slice Selection Gradients

- A slice rephasing lobe generally follows the slice-selection gradient.
- The slice selection gradient translates the band of frequencies into the desired band of locations.
- Increasing the amplitude of the slice selection gradient decreases the thickness of the slice for a fixed RF bandwidth



Mathematical description

$$f = \frac{\gamma}{2\pi} B \qquad \text{applying the slice selection gradient } \vec{\mathrm{G}}_{\mathrm{z}}$$

$$f = \frac{\gamma}{2\pi}(B_0 + G_z \Delta z)$$

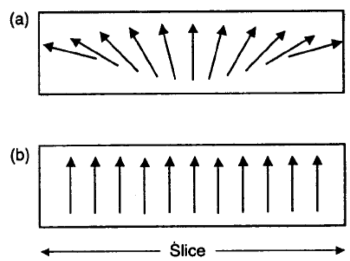
$$f_{rot} = \frac{\gamma}{2\pi} G_z \Delta z$$

$$\Delta z = \frac{2\pi \Delta f}{\gamma \vec{G}_z}$$

How can we obtain thinner slices??

Slice Rephasing

- Rephasing lobe restores the signal



Correction Gradients

Reference: Handbook of pulse sequence

Correction Gradients

1. Concomitant-Field Correction Gradients
2. Crusher Gradients
3. Eddy-Current Compensation
4. Spoiler Gradients
5. Twister Gradients

Concomitant-Field

- Gradients generated concurrently with the applied gradient produce magnetic field components perpendicular to B_z resulting in:
  - Deviating the net magnetic field vector from B_0
  - Causing the magnetic field to exhibit higher order spatial dependence know as Concomitant field
- The length of the Concomitant field is proportional to G^2/B_0
- Concomitant field occurs when a gradient is active and disappears when it is turned off

Concomitant-Field Phase

- Due to the Concomitant-Field the spins in the transverse plane accumulates phase that is spatially and temporally dependent
- Artifacts produced by the concomitant-field phase:
  - Geometric distortion, Image shift, ghosting, intensity loss, blurring, and shading

Concomitant-Field Phase Correction

- Concomitant-Field Phase correction during image reconstruction
- Hardware compensation
- **Alteration of the gradients waveforms in the pulse sequence**

Resulting Net Magnetic Field due to the Concomitant-Field

- Assuming cylindrical gradient coils:

$$B = B_0 + \vec{G}.\vec{r} + B_c$$
$$= B_0 + G_x x + G_y y + G_z z$$
$$+ \frac{1}{2B_0}\left[\frac{G_z{}^2}{4}(x^2 + y^2) + (G_x{}^2 + G_y{}^2)z^2 - G_x G_z xz - G_y G_z yz\right]$$

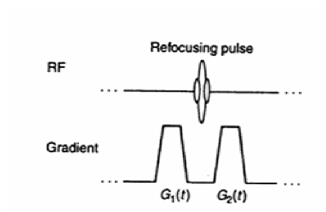
•*x, y, z* is the physical coordinate for the magnet, i.e. the magnet geometry play a role
•Squared term: self squared terms
•Hyperbolic terms: cross terms

Concomitant-Field Correction Gradient

- Achieved either by altering existing gradient lobes or by the addition of new gradient lobes.
- Several techniques are used:
  - **Waveform symmetrization**
  - Phase subtraction
  - **Waveform reshaping**
  - Quadratic nulling
  - Others

Waveform Symmetrization

- Negating the phase of a gradient lobe by adding an identical opposite lobe
- Examples: diffusion-weighting gradients

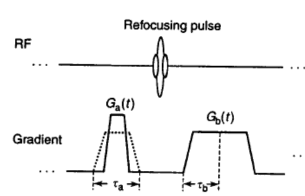


Waveform Reshaping

- The following two conditions must be satisfied:

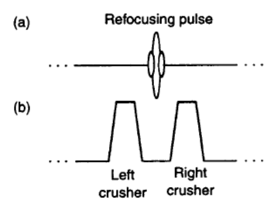
$$\int_0^{\tau_a} G_a(t)\,dt = \int_0^{\tau_b} G_b(t')\,dt'$$

$$\int_0^{\tau_a} G_a^2(t)\,dt = \int_0^{\tau_b} G_b^2(t')\,dt'$$

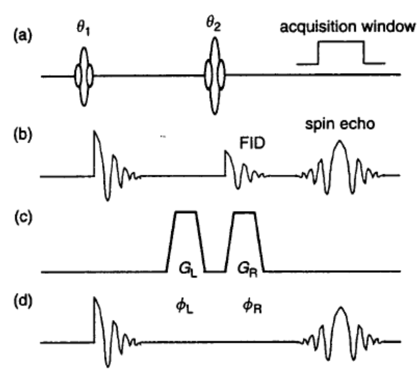


Crusher Gradients

- Crusher Gradients: is a correction gradient that preserves the desired signal pathways while eliminating unwanted ones by manipulating the phase of the signals.
- Consists of two lobes with the same polarity immediately before and after the refocusing RF pulse with the same or different areas.
- The crusher gradients are used to manipulate the phase coherence of the transverse magnetization.
- It can dephase or rephase the signal
- They are used with pulse sequences with at least one refocusing RF pulse



Quantitative Description



Qualitative Description

$$\phi_L(r) = \gamma A_L r$$
$$\phi_R(r) = \gamma A_R r$$

•For a spin-echo signal, the magnetization is in the transverse plane before and after the refocusing pulse. The net phase is zero if both areas are equal
•For the FID signal produced by by the nonideal refocusing pulse, only ϕ_R(r) applies
•If ϕ_R(r) is sufficiently large, the phase dispersion can completely destroys the signal coherence removing the FID from the data acquisition window.

Spoiler Gradients

- A spoiler gradient spoils unwanted signal that would otherwise produce artifact in the image.
- They are typically applied at the end of a pulse sequence.
- The longitudinal magnetization is preserved and experience no effect
- The area of the spoiler is large so it can adequately dephase the residual magnetization