Advanced MRI

Phase Difference Reconstruction

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Outline

- Introduction
- Quantitative Description
  - Arctangent operation
  - ATAN2
- Phased-Array Multiple Coil Data
- Correction of Predictable Phase Errors and the Concomitant Field
- Image Warping
- Image Scaling
- Noise Masking

Introduction

- MRI is a phase sensitive imaging modality
- Each pixel has a magnitude and phase data (complex nature of the MR signal). When standard magnitude reconstruction is performed phase information is discarded.**<u>phase info is not discarded but superimposed</u>**
- Information encoded in phase data
  - Bo homogeneity: used for Shimming
  - Fluid Flow: phase contrast angiography
  - MR temperature mapping
  - MR elastography

Introduction

- In MR acquisitions there are invariably unwanted contributions to the image phase.
  - System imperfections:
    - Gradient eddy currents
  - Unavoidable physical effects:
    - Chemical shift
    - Magnetic susceptibility variations
    - Concomitant field
- It is more difficult to interpret this phase map due to the unwanted phase. Two sets of acquisitions are made and a phase difference map is formed to address this problem.

Introduction

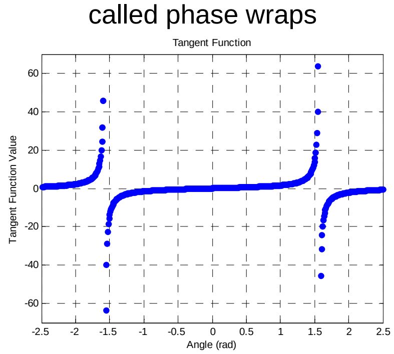
- Phase Difference Map: Difference between a pair of phase images on a pixel-by-pixel basis.
- The goal is to obtain the desired phase while eliminating the unwanted phase.

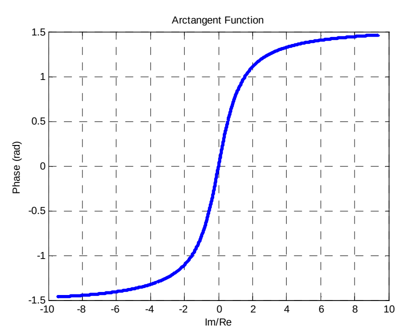
i.e. to produce a Bo map for shimming, two Gradient Echo data sets are acquired with identical parameters, except TE is varied. Difference of these two images gives the phase difference map.

i.e. in phase-contrast angiography: flow encoding gradient is modified between the two acquisitions and a phase diff map is reconstructed.

Quantitative Description

- Pixel-by-pixel arctangent operation
  - Output is defined in the interval $-\pi/2 < \arctan(x) < \pi/2$
  - Any values outside the primary range are represented by a value in the primary range (alias)
  - Aliasing in the phase map is accompanied by discontinuities called phase wraps





Quantitative Description

- Two k-space acquisitions to obtain a phase difference map
- The complex nature of MR signal
  - For a particular pixel on image 1➔ $z_1 = x_1 + iy_1 = \rho_1 e^{i\phi_1}$
    and image 2➔ $z_2 = x_2 + iy_2 = \rho_2 e^{i\phi_2}$

$$\Delta\phi = \phi_1 - \phi_2 = phase(z_1) - phase(z_2) = \arctan(\frac{y_1}{x_1}) - \arctan(\frac{y_2}{x_2})$$

- For computational efficiency and to minimize phase wraps, an optimal phase difference reconstruction should employ only a single arctangent operation per pixel, because two arctangent operations are computationally costly and introduce extra phase wraps.

$$\Delta\phi = \arg(\frac{z_1}{z_2}) = \arg(\frac{\rho_1 e^{i\phi_1}}{\rho_2 e^{i\phi_2}}) = \arg(\frac{\rho_1}{\rho_2} e^{i(\phi_1 - \phi_2)}) = \arctan(\frac{\mathrm{Im}(z_1/z_2)}{\mathrm{Re}(z_1/z_2)})$$

- Since complex conjugate and the inverse of a complex number z2 both have same phase

$$\Delta\phi = \arg(z_1 z_2^*) = \arg(\rho_1 e^{i\phi_1} \rho_2 e^{-i\phi_2}) = \arg(\rho_1 \rho_2 e^{i(\phi_1 - \phi_2)}) = \arctan(\frac{\mathrm{Im}(z_1 z_2^*)}{\mathrm{Re}(z_1 z_2^*)})$$

Quantitative Description

- Four Quadrant ATAN2 function
  - ATAN2 runs a test to check the sign of individual argument before forming their ratio

    i.e. num➔(+) & den➔(+), the phase lies in the 1st quadrant
    num➔(+) & den➔(-), ………………………2nd quadrant
    num➔(-) & den➔(-), ……………………….3rd quadrant
    num➔(-) & den➔(+), ………………………4th quadrant

- Thus the actual range of conventional arctangent function [-$\pi/2$, $\pi/2$] is extended to [-π, π]
- Still any phase difference value lies outside the primary range [-π, π] will be **aliased** back in the primary range. The true values can be obtained by adding or subtracting the multiples of 2π (phase unwrapping)
- Additional advantage of ATAN2 function is instead of reporting a divide-by-zero error when the second argument is zero it returns the correct value of -$\pi/2$, $\pi/2$

Phase Arranged Multiple Coil Data

- Many acquisitions use multiple coils arranged in a phased array.
- Optimal acquisition and processing of these data requires multiple receiver coils
- Using an algorithm a phase difference map can be reconstructed from this multichannel data.

$$\Delta\phi = \arg\left(\sum_j \frac{z_1 z_2^*}{\sigma_j^2}\right) = ATAN2\left(\mathrm{Im}\sum_j \frac{z_1 z_2^*}{\sigma_j^2}, \mathrm{Re}\sum_j \frac{z_1 z_2^*}{\sigma_j^2}\right)$$

Phase Arranged Multiple Coil Data

Example: suppose deltafi is near the alisaing boundary pi and there are 2 receiver channels . Because of noise

Delta fi1=pi-eps

Delta fi2=-pi+eps

Averaging these two phase gives simply zero. However if the multiple-receiver combination is made before the arctangent operation

$$\Delta\phi = \arg(z_1 z_2^*) = \arg\left(\sum \frac{\rho_{1j}\rho_{2j}e^{i\Delta\phi_j}}{\sigma_j^2}\right)$$

Assume the noise variance and image magnitudes have only weak dependence on the index j, so we can remove ρ and σ

$$= \arg(e^{i(\pi-\varepsilon)} + e^{i(-\pi+\varepsilon)}) = \arg(e^{i\pi}(e^{-i\varepsilon} + e^{-2\pi i}e^{i\varepsilon}))$$

$$= \arg(e^{i\pi}2\cos\varepsilon) = \pi$$

Correction of Predictable Phase Errors & The Concomitant Field

- After forming phase difference image, there are still unwanted phase errors
  - Phase contamination due to gradient eddy currents that can plague phase contrats angiography. Eddy currents are hardware dependent, so it is not very easy to predict the phase error in advance
  - However the constant and linear phase could be determined with a polynomial fit of the phase difference image where there should be no phase difference, such as stationary tissue in phase contrast angiogram.
    - Since the fitted phase is due entirely system imperfections, this can then be removed by post processing.

Correction of Predictable Phase Errors & The Concomitant Field

- Phase errors such as the one produced by concomitant field can be predicted with accuracy.
  - In that case it is advantageous to correct these phase errors before calculating the arctangent.
  - This avoids phase wraps in the phase difference map, and reduce the need for later phase unwarpping.
- Concomitant field has nonlinear spatial dependence, removing its phase error at this stage makes it easier to fit the eddy-current phase errors later.

Correction of Predictable Phase Errors & The Concomitant Field

- If the calculated concomitant field error at the pixel of interest is Fie, then the phase corrected version is

$$\Delta\phi_{corr} = \arg(z_1 z_2^* e^{-i\phi_e}) = ATAN2[\mathrm{Im}(z_1 z_2^* e^{-i\phi_e}), \mathrm{Re}(z_1 z_2^* e^{-i\phi_e})]$$

- For multiple coil case: the phase error from the concomitant field is independent of the receiver channel number, the phase correction for multiple coil case.

$$\Delta\phi_{corr} = \arg(e^{-i\phi_e} \sum_j \frac{z_{1j} z_{2j}^*}{\sigma_j^2})$$

- For further information on “Predictable Phase Errors and The Concomitant Field Phase Correction” refer to the journal article by Bernstein et al.1998

Image Warping

- Image warping operation can be applied to phase difference images.

Image Warping

- It is preferable to apply image warping prior to arctangent.
  - Since image warping is a conformal mapping that uses interpolation methods such as cubic splines, it may introduce phase wraps if applied after arctangent, and the sharp discontinuities can cause unwanted ringing in the image.
  - Also a phase difference image processed in this way cannot be properly unwrapped later, because the sharp transition between $-\pi$ and $+\pi$ has been distorted.
- If warping operation is indicated by function W

$$\Delta\phi_{warped} = \arctan\left(\frac{W(\mathrm{Im}(z_1 z_2^*))}{W(\mathrm{Re}(z_1 z_2^*))}\right) = ATAN2[W(\mathrm{Im}(z_1 z_2^*)), W(\mathrm{Re}(z_1 z_2^*))]$$

W should be on the entire image, not on a single pixel

- Again applying W before ATAN2 and separately to the real and imaginary images is advantageous because any change in image intensity caused by the warping algorithm will not effect the phase difference map.

Image Scaling

- The output of the ATAN2 function is real in the range of $-\pi$ and $+\pi$. sometimes we want to scale this output to a more convenient range by mutliplying it by some constant.

i.e.

In phase contrast angiography, the output can be scaled so that the pixel values represent velocity in convenient units such as millimeter per second.

For Bo mapping, the pixel intensity may represent the frequency offset δf in Hertz, tenths of Hertz or ppm.

Example

Suppose Bo map is obtained by forming the phase difference from the two GRE images with values of TE equal to 10ms and 25ms. How should the result of the phase difference map be scaled so that each intensity count represents a frequency offset of 0.1 Hz?

Solution: phase in radians $\Delta\phi$ ➔ frequency δf

$$\delta \mathrm{f}\left[\frac{\mathrm{Hz}}{10}\right] = \frac{10\Delta\phi}{2\pi\Delta\mathrm{TE}} = \frac{10\Delta\phi}{2\pi(0.025s - 0.010s)} = 106.1\Delta\phi$$

Noise Masking

- Noise
  - Standard magnitude image➔
  - Phase difference image➔
- Masking methods
- Threshold: a magnitude image is reconstructed from same data. M: the magnitude image corresponding to the phase difference map.

$$\mathrm{M} = \sqrt{\left|\sum_j \frac{z_1 z_2^*}{\sigma_j^2}\right|}$$

(for multiple coil case)

- Then the phase difference map can be masked

$$\Delta\phi_{thresh} = \begin{cases} \Delta\phi & M \geq M_0 \\ 0 & M < M_0 \end{cases}$$

- Multiplication: multiply the phase difference image by the magnitude image on a pixel by pixel basis.

$$\Delta\phi_{mult} = M\,\Delta\phi$$

Noise Masking

- Both methods have their advantages and disadvantages.

- The threshold method:
  - Phase difference pixel values are not altered, still some quantitative information can be found in the data
  - Over-aggressive method so can zero out interested pixel values. Finding proper threshold needs operator intervention.

- The multiplication method:
  - All pixel values are retained.
  - The masked data must be divided by the mask in order to obtain true phase difference information.

References