MR Overview (Cont'd)

- Last lecture
  - Main field B_0 and the larmor frequency
  - Radiofrequency field B_1 and the free induction decay (FID)
  - Linear gradient fields G and bandwidth
  - Quiz
    - G_x is turned on after an excitation and a signal is generated in a 1 T magnet. Assume G_x = 2G/cm, for a 20 cm wide object. What is the bandwidth (^{1}H with a gyromagnetic ratio of 42.58 MHz/T

- Required reading:
  - Ch1,2

MR Overview (Cont'd)

- The intrinsic magnetic properties of atomic nuclei form the basis of magnetic resonance.
- Atomic nuclei such as, 1H, 13C, 23Na, 31P, etc., which are composed of an odd number of protons or neutrons, possess a net nuclear angular momentum and a magnetic moment.

MR Overview (Cont'd)

- By placing a proton in a static external magnetic field , it precesses around the axis of that field at a frequency proportional to the strength of the external magnetic field.
- The precessional frequency , known as the Larmor frequency, is governed by

$$\omega_0 = \gamma B_0$$

Bloch Equation

- In the presence of a magnetic field , the synchronous precession of the collection of protons in the sample produces a net magnetization vector given by the Bloch equation.
- Bloch equation:
  - vector equation
  - The relaxation terms describe the return to equilibrium for a field pointing along the *z*-axis

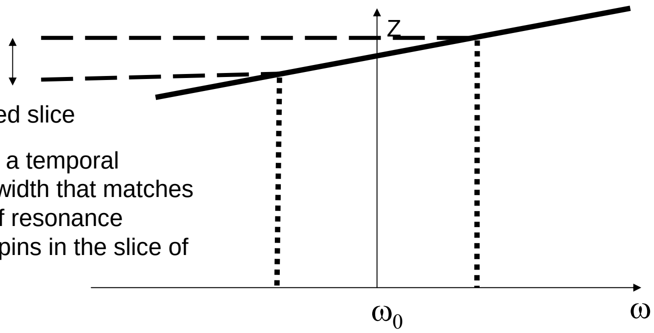
$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{ext} - \frac{M_x \hat{i} + M_y \hat{j}}{T_2} - \frac{(M_z + M_0)\hat{k}}{T_1}$$

MR Imaging Methods

- Imaging is divided into excitation and reception.
- Selective excitation: it is achieved by applying B_1 in the presence of B_0 and G.
- Applying an additional transverse on-resonance field, i.e., at the Larmor frequency,

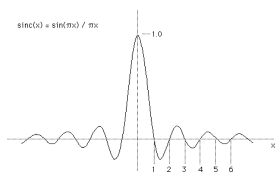
Slice selection

ΔZ

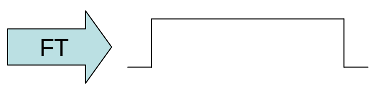


Excited slice

B1 must posses a temporal frequency bandwidth that matches the bandwidth of resonance frequencies of spins in the slice of interest

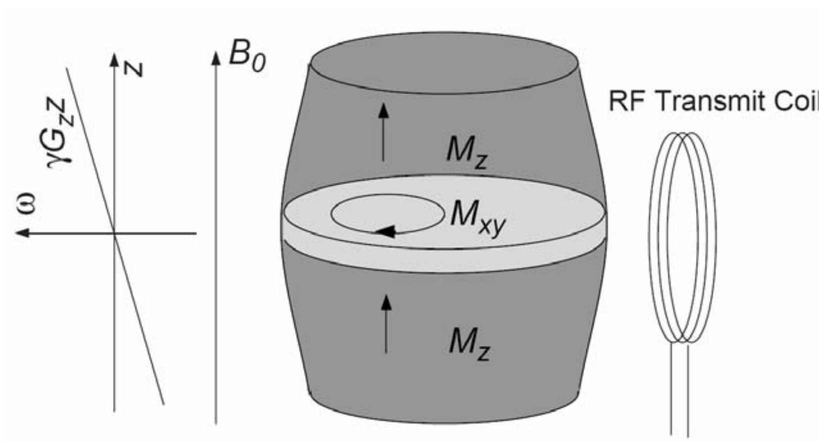


$$\vec{B}_1 = B_1(t)\cos\omega_0 t\ \hat{i} - B_1(t)\sin\omega_0 t\ \hat{j}$$

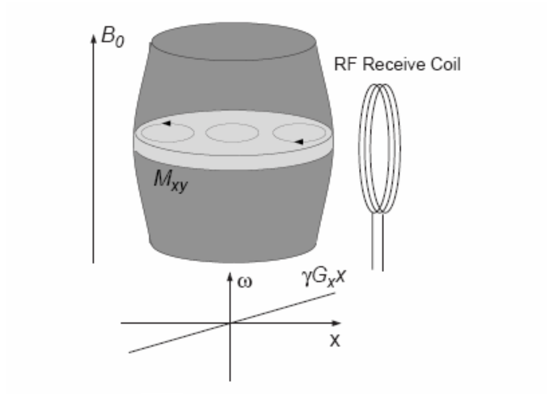


Bandwidth= $\gamma\ G_z\ \Delta Z$

$B_1(t)$



A magnetic field with a gradient in the z direction establishes a linear relationship between position and resonant frequency. When the sample is irradiated with an RF pulse at the Larmor frequency B0, only the spins at z = 0 are on resonance, and are tipped into the transverse plane. Hence, only a slice is excited.



A magnetic field with a gradient in the x direction ,again, establishes a linear relationship between position and resonant frequency. Spins at different x positions precess at different frequencies.

- Required reading:
  section 3.2.2

Classical description of nuclear spins

- Atoms with odd number of protons/ neutrons possess a spin angular momentum

$$S = \hbar I$$

- S: spin angular momentum
- h/2π
- I: spin operator

$$\mu = \gamma S$$

- μ : magnetic dipole moment

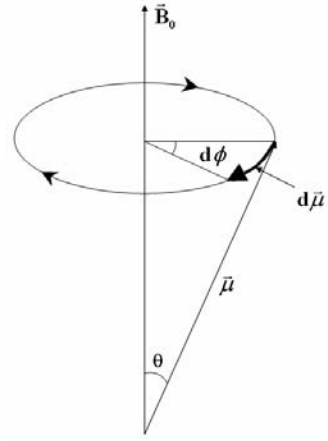
μ measured in a unit called the nuclear magneton (Joule/Tesla)

Equation of Motion

- In the presence of an external magnetic field, the spins possessing magnetic dipole moment vector in the direction of the spins axis will precess about the static magnetic field direction obeying the equation of motion given by (simple version of the bloch equation

$$\frac{d\vec{\mu}}{dt} = \gamma \vec{\mu} \times \vec{B}_0$$

Spin precession about the static magnetic field

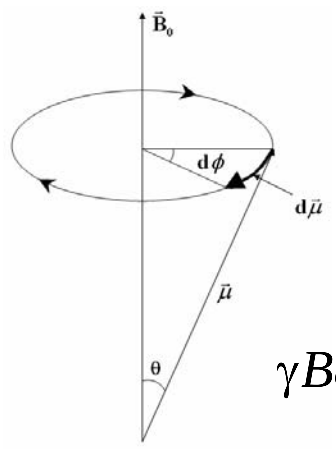


The interaction of the proton's spin with the magnetic field produces the torques, causing it to precess about the main magnetic field

Precision via the gyroscope analogy

- The precession frequency of the spin around the static magnetic field can be derived with the aid of the equation of motion

$$\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu} \mathrm{x} \vec{B}_0$$



$$\left|d\vec{\mu}\right| = (\mu \sin\theta)\left|d\phi\right|$$

$$\left|d\vec{\mu}\right| = (\gamma\mu B \sin\,\theta)dt$$

$$\gamma B dt \;= \left|d\phi\right| \Rightarrow \frac{d\phi}{dt} \equiv \omega = -\gamma$$

- The Larmor frequency is a constant for a given static magnetic field. The magnetization vector in a volume *V* is defined as

$$\vec{M} = \frac{1}{V}\sum_{\substack{Spins \\ in \quad V}} \vec{\mu}_i$$

- Applying an additional transverse on-resonance field created from a radio frequency (RF) coil (by "on-resonance" meaning it oscillates at the Larmor frequency), given by

$$\vec{B}_1 = B_1(t)\cos\omega_0 t\ \hat{i} - B_1(t)\sin\omega_0 t\ \hat{j}$$

results in a time-dependent behavior of M

$$\frac{dM_x}{dt} = \gamma(M_y B_0 + M_z B_1 \sin\omega_0 t)$$

$$\frac{dM_y}{dt} = \gamma(M_z B_1 \cos\omega_0 t - M_x B_0)$$

$$\frac{dM_z}{dt} = -\gamma(M_x B_1 \sin\omega_0 t + M_y B_1 \cos\omega_0 t)$$

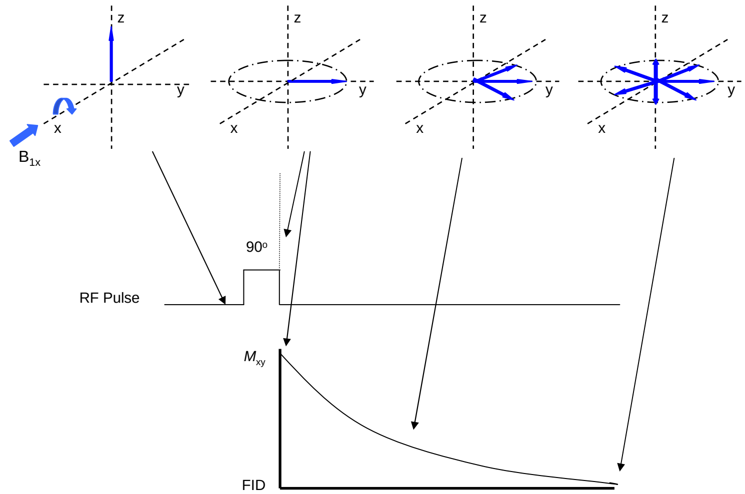
- Following the RF excitation, the time-dependent magnetization returns to equilibrium, a process that can also be represented by Bloch equations

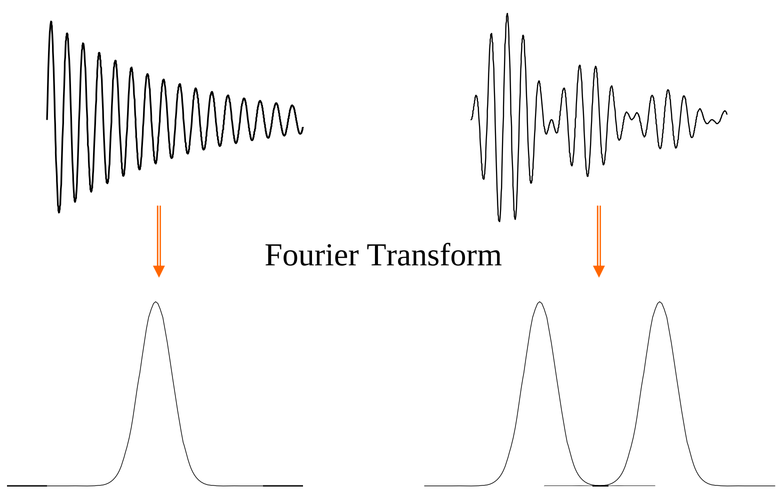
$$\frac{dM_x}{dt} = \omega_0 M_y - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = -\omega_0 M_x - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1}$$

Free induction decay (FID) of magnetization after a 90o pulse.





- Draw the sinc function and obtain its FFT using matlab
- Draw 300 cosines with random frequencies